

$$\textcircled{1} \text{ a) } 2\sqrt{45} - 3\sqrt{125} + \frac{3}{2}\sqrt{20} - \frac{5}{4}\sqrt{80} =$$

$$= 2\sqrt{3^2 \cdot 5} - 3\sqrt{5^3} + \frac{3}{2}\sqrt{2^2 \cdot 5} - \frac{5}{4}\sqrt{2^3 \cdot 5} = 6\sqrt{5} - 15\sqrt{5} + 3\sqrt{5} - 5\sqrt{5} = -11\sqrt{5}$$

$$\text{b) } \frac{3+2\sqrt{2}}{3-2\sqrt{2}} \cdot \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{9+8+12\sqrt{2}}{9-8} = 17+12\sqrt{2}$$

$$\text{c) } (\sqrt{3}+\sqrt{2})^2 - (\sqrt{3}-\sqrt{2})^2 - (\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) =$$

$$= 3+2+2\sqrt{6} - (3+2-2\sqrt{6}) - (3-2) = 5+2\sqrt{6} - 5+2\sqrt{6} - 1 = 4\sqrt{6} - 1$$

$$\textcircled{2} \text{ a) } \log_2 \sqrt[3]{\left(\frac{1}{64}\right)^{-5}} = \log_2 \sqrt[3]{(2^{-6})^{-5}} = \log_2 \sqrt[3]{2^{30}} = \log_2 2^{\frac{30}{3}} = \log_2 2^{10} = 10$$

$$\text{b) } \log_3 x = 0,85; \log_3 100x - \log_3 \frac{x}{1000} = \log_3 100 + \log_3 x - (\log_3 \sqrt[3]{x} - \log_3 1000) =$$

$$= 2 + \log_3 x - \left(\frac{1}{3} \log_3 x - 3\right) = 5 + \frac{2}{3} \log_3 x =$$

$$= 5 + \frac{2}{3} \cdot 0,85 = 5,57$$

$$\textcircled{3} \text{ a) } \sqrt{4x+1} - 2x+5 = x+2 \Rightarrow \sqrt{4x+1} = 3x-3 \Rightarrow 4x+1 = 9x^2 - 18x + 9 \Rightarrow$$

$$\Rightarrow 9x^2 - 22x + 8 = 0 \Rightarrow 9x^2 - 4x - 18x + 8 = 0 \Rightarrow x(9x-4) - 2(9x-4) = 0$$

$$\Rightarrow (9x-4)(x-2) = 0 \Rightarrow \begin{cases} x=2 & \text{Si es solución } 4=4 \\ x=\frac{4}{9} & \text{No es solución } \sqrt{\frac{16}{9}+1} - \frac{8}{9} + 5 = \frac{5}{3} \cdot \frac{8}{9} + 5 = \frac{52}{9} \end{cases}$$

$$\text{b) } \frac{2x}{x^2-1} - \frac{x+1}{x-1} = -\frac{5}{4} \Rightarrow \frac{2x - (x+1)(x+1)}{(x^2-1)} = -\frac{5}{4} \quad \frac{4}{9} + 2 = \frac{22}{9} \neq$$

$$\frac{2x - x^2 - 2x - 1}{x^2-1} = -\frac{5}{4} \Rightarrow -4x^2 - 4 = -5x^2 + 5 \Rightarrow x^2 = 9$$

$$\Rightarrow \boxed{x = \pm 3}$$

$$c) \begin{cases} 2x+y+z=0 \\ x+2y-z=-3 \\ 3x+2y=-1 \end{cases} \left(\begin{array}{ccc|c} z & x & y & \\ 1 & 2 & 1 & 0 \\ -1 & 1 & 2 & -3 \\ 0 & 3 & 2 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 3 & 3 & -3 \\ 0 & 3 & 2 & -1 \end{array} \right) \sim$$

$F_2 \rightarrow F_1 + F_2$

$F_3 \rightarrow F_2 - F_3$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 1 & -2 \end{array} \right) \begin{matrix} z+2x+y=0 \\ x+y=-1 \\ y=-2 \end{matrix} \begin{matrix} z = -2x - y = -2 + 2 = 0 \\ x = 1 \\ y = -2 \end{matrix} \begin{matrix} x=1 \\ \text{S.C.D. } y=-2 \\ z=0 \end{matrix}$$

4- a) $3^{x+1} + 3^{x-2} = \frac{28}{3} \Rightarrow 3^x \cdot 3 + \frac{3^x}{3^2} = \frac{28}{3} \Rightarrow 3y + \frac{y}{9} = \frac{28}{3} \Rightarrow$
 $\Rightarrow 27y + y = 84 \Rightarrow 28y = 84 \Rightarrow y = 3 \Rightarrow 3^x = 3 \Rightarrow \boxed{x=1}$

b) $\frac{\log_2(4x-3) + \log_2 6}{\log_2(5x+1)} = 1 \Rightarrow \log_2(4x-3) + \log_2 6 = \log_2(5x+1) \Rightarrow$
 $\Rightarrow \log_2 6 \cdot (4x+3) = \log_2(5x+1) \Rightarrow 24x+18 = 5x+1 \Rightarrow 19x = 19 \Rightarrow \boxed{x=1}$

Es válida

c) $4^{3x-1} = \left(\frac{1}{2}\right)^{2x} \Rightarrow (2^2)^{3x-1} = (2^{-1})^{2x} \Rightarrow 2^{6x-2} = 2^{-2x} \Rightarrow$
 $\Rightarrow 6x-2 = -2x \Rightarrow 8x = 2 \Rightarrow \boxed{x = \frac{1}{4}}$

d) $7^{x-3} = 14 \Rightarrow \log_7 7^{x-3} = \log_7 14 \Rightarrow (x-3) \log_7 7 = \log_7 14 \Rightarrow$

$$\Rightarrow x-3 = \frac{\log_7 14}{\log_7 7} \Rightarrow x = 3 + \frac{\log_7 14}{\log_7 7} = 3 + \frac{\log_7 7 \cdot 2}{\log_7 7} = 3 + \frac{\log_7 2}{\log_7 7} = 3 + 1 + \frac{\log_7 2}{\log_7 7} = 4 + \frac{\log_7 2}{\log_7 7}$$

5- a) $-x^2 - x + 6 \geq 0 \Rightarrow -x^2 - x + 6 = 0 \Rightarrow x^2 + x - 6 = 0$
 $\Rightarrow x \in [-3, 2]$ $x = -3 \quad x = +2$

b) $\frac{x(x^2+1)}{(x+3)(x-2)} \geq 0$

$x=0$
 $x^2+1 \neq 0$
 $x+3=0 \Rightarrow x=-3$
 $x-2=0 \Rightarrow x=2$

$x \in (-3, 0] \cup (2, +\infty)$