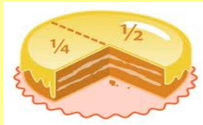
UNIT 4 VOCABULARY: FRACTIONS

0. Introduction

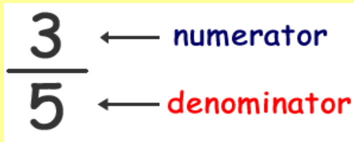
A fraction is a number that expresses part of a unit or a part of a quantity.

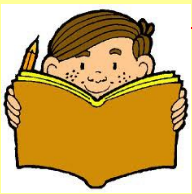


Fractions are written in the form $\frac{a}{b}$ where a and b are integers, and the number b is not 0.

A fractions has **two parts**:

- The top number ***a*** is called the **numerator**.
- The number ***b*** is called the **denominator**.





READING FRACTIONS!!!

We use the **cardinals to name the numerator** and the **ordinals for the denominator**.

For example: $\frac{2}{3}$ --> two thirds $\frac{7}{5}$ --> seven fifths $\frac{1}{8}$ --> one eight

However, there are **three exceptions**:

- **When the denominator is 2, it is read "half".**

  For example: $\frac{1}{2}$ --> one **half** $\frac{3}{2}$ --> three **halves**
- **When the denominator is 4, it can be read as "fourth" or "quarter".**

  For example: $\frac{1}{4}$ --> a fourth or a **quarter** $\frac{3}{4}$ --> three **quarters** or three fourths
- **For denominators greater than 10, we say "over" and do not use ordinal.**

  For example: $\frac{12}{15}$ --> twelve **over** fifteen $\frac{17}{32}$ --> seventeen **over** thirty-two

BE CAREFUL WITH PLURALS!

If the numerator is greater than 1, you must use plurals with ordinals.

$\frac{5}{2}$ --> five **halves** $\frac{3}{4}$ --> three **quarters** $\frac{7}{10}$ --> seven **tenths**

Exercise. Write in words and read the following fractions:

$\frac{11}{4}$ ________________	$\frac{3}{8}$ ________________	$\frac{9}{6}$ ________________
$\frac{17}{2}$ ________________	$\frac{4}{9}$ ________________	$\frac{19}{62}$ ________________
$\frac{1}{12}$ ________________	$\frac{14}{93}$ ________________	$\frac{17}{100}$ ________________

1.1 Fractions as a division

A fraction can express **a division**. The **numerator** is the **dividend** and the **denominator** is the **divisor**.

For example:

$$\frac{4}{2}=4\div 2=2 \qquad \frac{3}{4}=3\div 4=0.75 \qquad \frac{53}{100}=0.53$$

Exercise. Divide:

a) $\frac{7}{2}$ b) $\frac{9}{10}$ c) $\frac{5}{4}$ d) $\frac{5}{8}$

1.2. Fraction as part of a whole

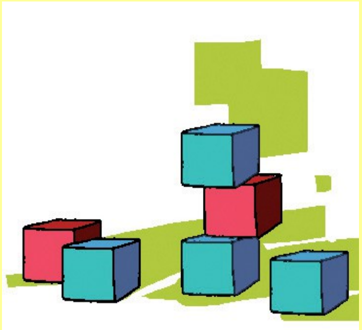
- The **numerator** tells us how **many equal parts we have**.
- The **denominator** tells us **how many equal parts are available**.

For example:

What fraction of these cubes are red? → $\frac{2}{6}$

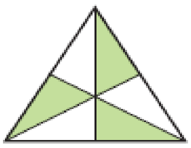
What fraction of these cubes form a column? $\frac{3}{6}$

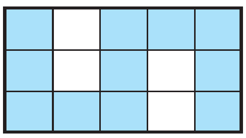
What fraction of the blue cubes form a column? $\frac{2}{4}$

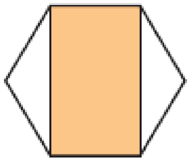


ALL PARTS MUST BE EQUAL!!!!!

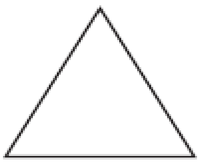
Exercise. Write and read the fractions that represent the shaded portions.

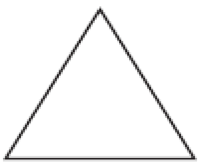


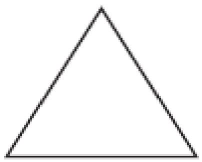




Exercise. Represent each fraction on this triangles:

$\frac{1}{2}$ 

$\frac{2}{3}$ 

$\frac{3}{4}$ 

1.3. Fraction as an operator

A fraction is also **an operator**. You just **multiply by the numerator** and **divide by the denominator**.

For example, to find out 2/3 of 12 eggs, you just operate:
(12 · 2) ÷ 3 = 24 ÷ 3 = 8 eggs.

Exercises.

1. Find out:

a) $\frac{5}{8}$ of 400 **b)** $\frac{4}{5}$ of 20 **c)** $\frac{5}{6}$ of 60

2. Fill in the gap:

a) $\frac{3}{4}$ of ____ are 15 **b)** $\frac{2}{3}$ of _____ are 40 **c)** $\frac{4}{5}$ of _____ are 20

1.4. Proper and improper fractions

- A fraction is **proper** if the denominator is greater than the numerator.
- A fraction is **improper** if the numerator is grater than the denominator.
- A fraction is **a whole (a unit)** if the denominator is equal to the numerator.

1.6 Sign of a fraction

Each term of a fraction can be positive or negative. You can find four cases (but there are **just two** indeed):

- If both numerator and denominator have **the same sign**, the fraction is **positive**. For example:

$$\frac{+3}{+5}=\frac{3}{5} \qquad \frac{-2}{-7}=\frac{2}{7}$$

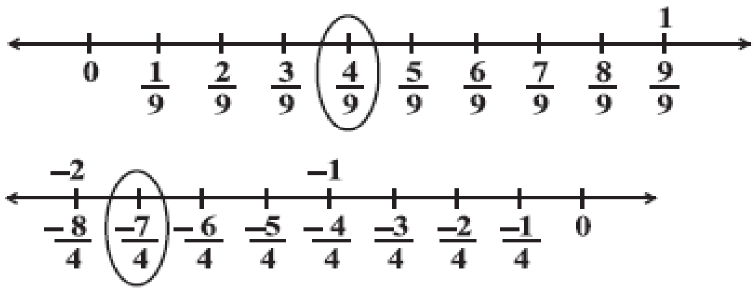
- If both numerator and denominator have **different signs**, the fraction is **negative**. For example:

$$\frac{+4}{-9}=-\frac{4}{9} \qquad \frac{-6}{+5}=-\frac{6}{5}$$

1.7 Representation of fractions on the number line

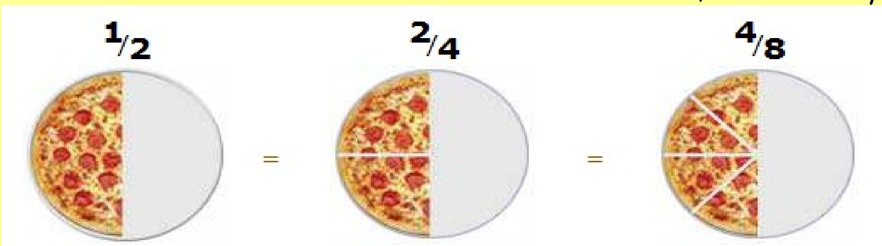
Any fraction can be represented on the number line. In a fraction, the denominator tells us the number of equal parts into which the first unit has been divided; the numerator tells us 'how many' of these parts are considered.

So, a rational number such as $\frac{7}{3}$ means four of nine equal parts ***on the right of 0***, and for $-\frac{7}{4}$ we make 7 markings of distance $\frac{1}{4}$ each ***on the left of zero*** and starting from 0.

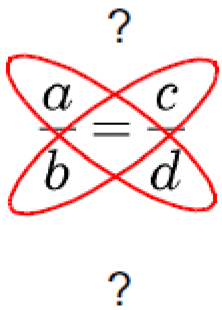


2.1. Equivalent fractions

Equivalent fractions are fractions that look different from each other, but are really **the same**.



We can test if two fractions are equivalent by **taking the cross-product,** this is, two fractions are equivalent if



$$a \cdot d = b \cdot c$$

For example, if we want to test if $\frac{20}{12}$ and $\frac{40}{24}$ are equivalent fractions:

- **The first cross-product** is the product of the first numerator and the second denominator: 12 × 40 = 480.
- **The second cross-product** is the product of the second numerator and the first denominator: 24 × 20 = 480.

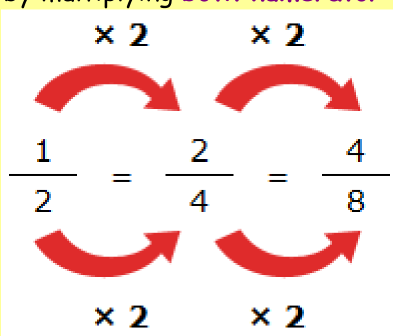
The cross-products **are the same, so the fractions are equivalent.**

Exercise. Test if these fractions are equivalent fractions:

a) $\frac{3}{7}$ and $\frac{18}{42}$ **b)** $\frac{2}{4}$ and $\frac{13}{20}$

2.2. Making equivalent fractions

You can **make equivalent fractions** by multiplying **both numerator and denominator** by the same amount.



Exercise. Write a sequence of equivalent fractions as in the example:

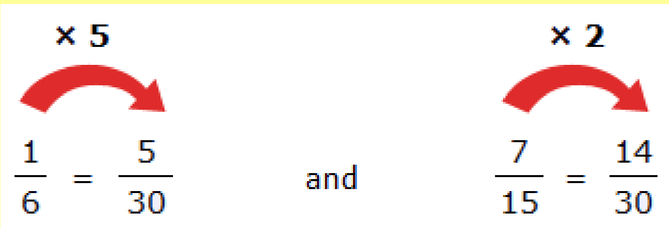
Starting fraction	Equivalent fractions					
$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$	$\frac{7}{21}$
$\frac{2}{5}$						
$\frac{8}{16}$						
$\frac{7}{3}$						

2.3. Least Common Denominator

When working with fractions, sometimes **we need the denominators of two fractions to be the same. We can use equivalent fractions to do that.**

For example, let's write $\frac{+3}{+5}=\frac{3}{5}$ and $\frac{7}{15}$ with a common denominator.

- **What is the new denominator?**
  The new denominator is the Lowest Common Multiple.
  multiples of 6 ➔ 6, 12, 18, 24, **30**, 36, ... **multiples 15** ➔ 15, **30**, 45, 60, ...
  LCM (6, 15) = 30.
- **What is the new numerator?**
  We divide every new denominator by the previous one and we multiply the result by the numerator.
  30÷6 = 5 **30÷15 = 2**



And that's all!!!

2.4. Comparing and ordering fractions

REMEMBER! SYMBOLS

To compare two numbers, we can use these symbols:

Symbol	Is read	Example	Is read
=	**Is equal to / equals**	$\frac{1}{2}=\frac{2}{4}$	A half equals two quarters
≠	**Isn't equal to / doesn't equal / is different from**	$\frac{2}{3}\neq\frac{3}{2}$	Two thirds doesn't equal three halves

$<$	Is less than	$\frac{2}{3}<\frac{3}{4}$	Two thirds is less than three quarters
$>$	Is greater than	$\frac{5}{3}>\frac{5}{6}$	Five thirds is greater than five sixths

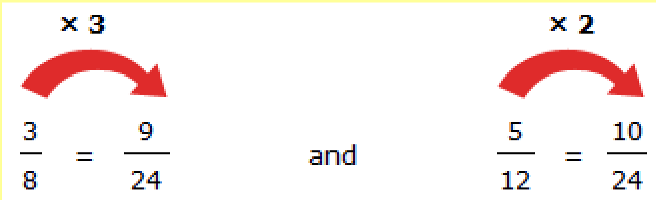
- If two fractions have the **same denominator** then they are easy to compare. Look at their numerators, and the largest fraction is the one with the largest numerator.

  For example, $\frac{4}{9}$ is less than $\frac{5}{9}$ (because 4 is less than 5)

  $$\frac{4}{9}<\frac{5}{9}$$

- If the **denominators are not the same,** you need to make them the same (see the previous section).

  For example, $\frac{3}{8}$ and $\frac{5}{12}$



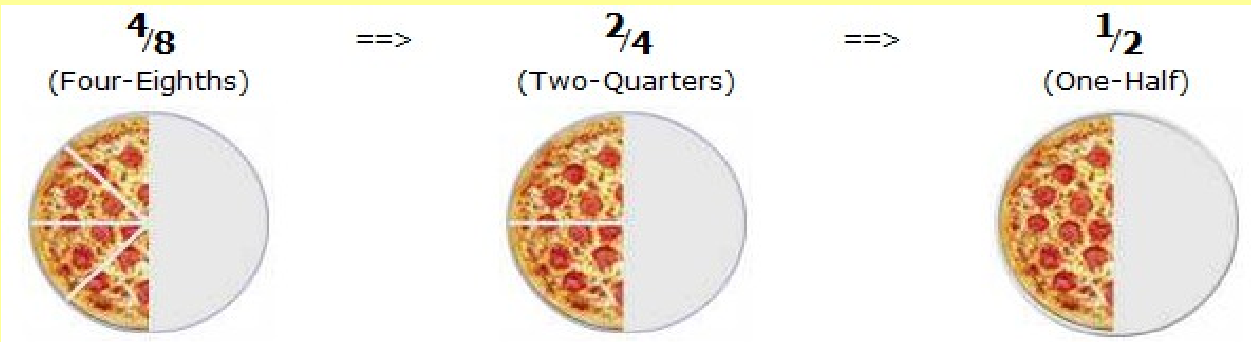
So $\frac{3}{8}<\frac{5}{12}$

Exercise. For each fraction pair, put them both over a common denominator to see which is bigger.

$\frac{3}{4}$ and $\frac{4}{5}$ $\frac{1}{3}$ and $\frac{2}{5}$ $\frac{13}{20}$ and $\frac{7}{10}$

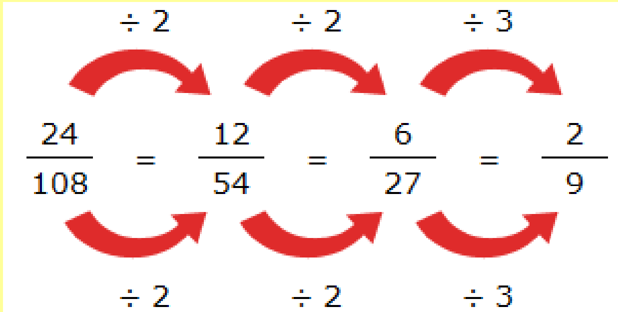
2.5. Simplest form of a fraction

Simplifying (or reducing) a fraction means to make the fraction as simple as possible.



To find the simplest form of a fraction, **try dividing both the top and bottom of the fraction until you can't go any further** (try dividing by 2,3,5,7,... etc).

For example, to reduce $\frac{24}{108}$ to its lowest terms:



Exercises.

1. Find the simplest form of these fractions:

   $\frac{24}{36}=$ $\qquad \frac{75}{55}=$ $\qquad \frac{84}{240}=$ $\qquad \frac{50}{120}=$

2. Express, in the simplest form, which fraction corresponds to these situations:
   - **a)** In a bag of 90 pens, 15 are blue.
   - **b)** The number of girls and boys in our class.
   - **c)** There are 90 pupils of the 270 who come by bus to the school.

3.1. Addition and subtraction with fractions

REMEMBER! + → PLUS - → MINUS

To add or subtract fractions:

- **If the fractions have the same denominator,** the numerator of the sum or the difference is found by simply adding or subtracting the numerators over the denominator.

For example, $\frac{2}{4}+\frac{3}{4}=\frac{2+3}{4}=\frac{5}{4}$ $\qquad \frac{2}{4}-\frac{3}{4}=\frac{2-3}{4}=-\frac{1}{4}$

REDUCE ALWAYS WHEN POSSIBLE !!!!

- **If the denominators are different,** follow these steps:
  - Reduce them to a **common denominator** (see comparing fractions).
  - **Add or subtract the numerators** and do not change the denominator.
  - **Reduce** (if possible).

For example, to find out $\frac{5}{6}-2+\frac{9}{4}$. The LCM of 6 and 4 is 12, so

$$\frac{5}{6}-2+\frac{9}{4}=\frac{10-24+27}{12}=\frac{13}{12}$$

Exercise. Work out:

a) $\frac{3}{8}+\frac{1}{4}+\frac{3}{16}$ **b)** $2+\frac{1}{9}-\frac{3}{5}$ **c)** $\frac{4}{7}-\frac{2}{3}$ **d)** $\frac{1}{2}+\frac{1}{3}-\left(\frac{1}{4}+\frac{1}{5}\right)$ **e)** $\frac{1}{4}-\frac{1}{8}-\left(\frac{1}{3}+\frac{1}{6}\right)$

4.1. Multiplication of fractions

REMEMBER! · → TIMES/MULTIPLIED BY

To multiply fractions:

- Multiply the top numbers (numerators).
- Multiply the bottom numbers (denominators).
- Simplify the fraction (if possible).

$$\frac{1}{3} \times \frac{9}{10} = \frac{1 \times 9}{3 \times 10} = \frac{9}{30} = \frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

Exercise. Work out:

a) $\frac{3}{2} \cdot \frac{4}{9}$ **b)** $\frac{2}{3} \cdot \frac{1}{4} \cdot \frac{3}{5}$ **c)** $15 \cdot \frac{2}{5}$ **d)** $\frac{4}{3} \cdot 5$ **e)** $\frac{1}{9} \cdot \frac{3}{11} \cdot \frac{4}{7}$

4.2. Division with fractions

REMEMBER! : → DIVIDED BY

To divide fractions:

- Multiply the numerator of the first fraction by the denominator of the second.
- Multiply the denominator of the first fraction by the numerator of the second.
- Reduce the fraction (if possible).

$$\frac{1}{3} \div \frac{4}{5} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$$

Exercise. Work out:

a) $\frac{4}{5} : \frac{3}{7}$ **b)** $\frac{9}{12} : 7$ **c)** $\frac{\frac{3}{7}}{\frac{2}{8}}$ **d)** $\left(\frac{5}{3} \div \frac{5}{8}\right) \div \frac{7}{4}$

1.4. Order of Operations

When you have several operations to do, which one do you calculate first?

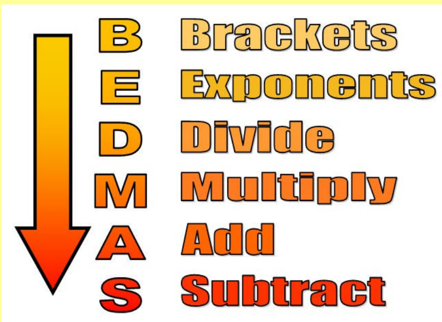
We work out operations in this order:

BRACKETS

EXPONENTS (Powers, roots, etc)

DIVISION and **M**ULTIPLICATION (working from left to right)

ADDITION and **S**UBTRACTION (working from left to right)



That makes **BEDMAS!**

Exercise. Work out these operations:

a) $\frac{5}{2}\cdot\frac{1}{2}-\frac{4}{3}$ **b)** $\frac{1}{5}+\frac{2}{3}\div\frac{5}{7}$ **c)** $\frac{1}{4}\cdot\frac{2}{3}+\frac{1}{6}\cdot\frac{3}{2}$ **d)** $\left(\frac{1}{2}+\frac{2}{5}\right)\div\left(1-\frac{1}{10}\right)$