UNIT 3 VOCABULARY: INTEGERS

3.1. Some uses of negative numbers

There are many situations in which you need to use negative numbers.

- **POSITIONS**

A submarine which is sailing 700 m below sea level		- 700 m
The height of Mount Everest is 8700 m above sea level		+ 8,700 m
The second floor of a subterranean garage		-2

- **MONEY**

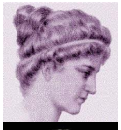
Peter earns € 5000.	+ 5,000 €
Mary spends £ 200 in clothes	- 200 £
Susan saves $ 700 in his bank account	+ 700 $
Yoshiro is ¥ 40,000 overdrawn	- 40,000 ¥

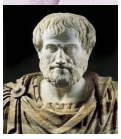
- **TEMPERATURE**

The average summer temperature here is 35 °C	+ 35 °C
The temperature in Siberia has risen 25 °C below zero	- 200 °C
The temperature goes up 5 °C	+ 5 °C
The temperature goes down 3 °C	- 3 °C

- **YEARS**

Hypatia of Alexadria (Egypt) was a Greek philosopher and mathematician. Hypatia lived in Roman Egypt, and was tortured and murdered in **AD 415**	+ 415
Aristotle was a very famous Greek philosopher, a student of **Plato** and teacher of **Alexander the Great**. He was born in Stageira, Chalcidice, in 384 BC.	- 384





Exercise. Write an integer to represent each situation:

Eleven degrees above zero	**A loss of € 27**	**Christopher Columbus discovers America in AD 1492**
Aristotle died in 322 BC	**The temperature drops 5°C**	**Five degrees above zero**
Earnings of €200	**A submarine is 500 m below sea level**	**The temperature goes up 9 °C**

1.3. The integers set

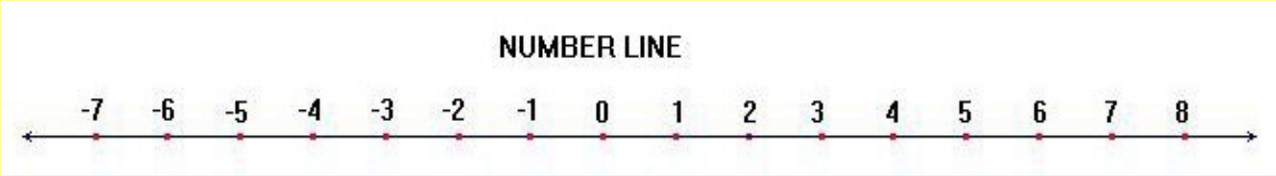
In Mathematics, **the integer set** is formed by the natural numbers, zero and negative numbers. It is represented by $\mathbb{Z}$.

$$\mathbb{Z} = \{\ldots\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\ldots\}$$

The integer set doesn't have a beginning or an ending and it has infinite elements.

2.1. The Number Line

The number line is a line labelled with the integers in increasing order from left to right, that extends in both directions:



TWO IMPORTANT THINGS:

- Negative numbers are on the left
- Positive numbers are on the right

Exercise. Represent on a line the following numbers: -1, +3, +7, -5

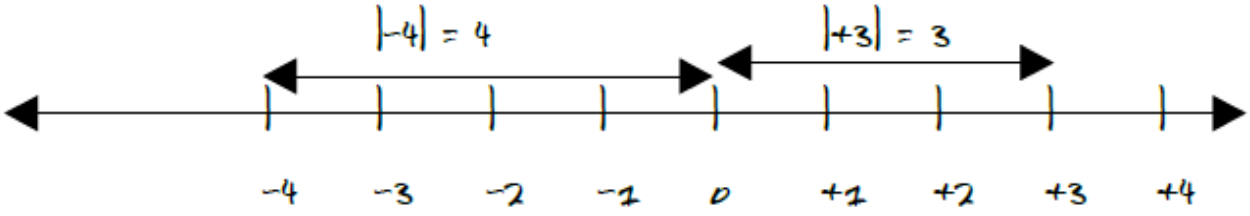
2.2. Absolute Value

The absolute value of an integer is the number without the sign. It is represented as **|a|** and it is called absolute value of a.

For example:

|+5| = 5 |-3| = 3 |-7| = 7 |+2| = 2

The absolute value represents **how far** a number is from zero:



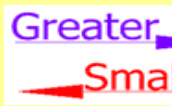
Exercises.

1. Work out the absolute value of these numbers:
   a) |-9| **b)** |+5| **c)** |-3| **d)** |+7| **e)** |0|

2. Find out the number that has an absolute value of 7 and is between –8 and -6.

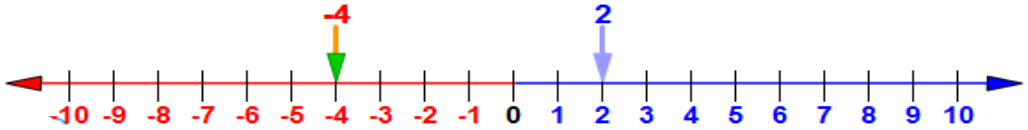
2.3. Order in the integer set

For any two different places on the number line, the integer on the right is greater than the integer on the left.

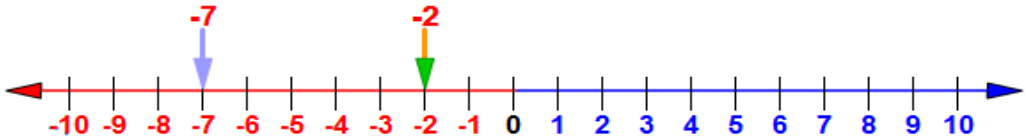


NUMBERS ON THE LEFT ARE SMALLER THAN NUMBERS ON THE RIGHT

For example:



-4 < 2



-7 < -2

Remember how to read the different symbols!!!!

Symbol	Is read	Example	Is read
=	Is equal to / equals	-5 = -5	Minus five equals minus five
≠	Isn't equal to / doesn't equal	-2 ≠ 2	Minus two doesn't equal two
<	Is less than	-4 < -2	Minus four is less than minus two
≤	Is less than or equal to	5 ≤ 5 -5 ≤ 2	Five is less than or equal to five Minus five is less than or equal to two
>	Is greater than	0 > -6	Zero is greater than minus six
≥	Is greater than or equal to	-1 ≥ -1 -1 ≥ -2	Minus one is greater than or equal to minus one Minus one is greater than or equal to minus two

Exercises.

1. Write down the "less than" sign (<) or the "greater than" sign (>) where corresponds:
   a) +4___+1 **b)** -1___- 6 **c)** 0 ___+3 **d)** -8___+2 **e)** -2___0

2. Sort these negative numbers, greatest first: -5, -1, -2, -25.

3. Arrange in ascending order:
   a) -12, +3 , 0, -1, -5, 5, +7, -18, 4
   b) 11, -7 , -3, +1, -10, 8, +13, -15, 2

4. Arrange in descending order:
   a) -11, +7 , 10, -1, -5, 15, -7, -15, 4
   b) 11, -17 , +3, -1, 0, 8, +13, -18, 2

2.4. Opposite

The opposite of an integer is another integer with the same absolute value but different sign.

For example:
Op (+3) = -3 Op (-5) = +5 Op (-7) = +7 Op (+2) = -2

Exercises.

1. Work out the opposite of the following numbers:
   a) Op (-4) = **b)** Op (+8) = **c)** Op (15) = **d)** Op (-301) =

2. Complete this chart.

Number	Absolute value	Opposite	One less than	One more than
0				
5				
-7				
-1				

4.1. Addition of integers

- When **adding integers with the same sign**, we add their absolute values, and give the result the **same sign**.
  For example:
  5 + 3 = 8 2 + 7 = 9 (-2) + (-7) = -9 -5 + (-3) = -8
- When **adding integers with opposite signs**, we take their absolute values, subtract the smaller from the larger, and give the result the sign of the integer with the **larger** absolute value.
  For example:
  7 + (-2) = 5 2 + (-3) = -1 - 5 + 7 = 2 -3 + 1 = -2

Exercises.

1. Work out:
   a) -8 + (-3) = **b)** 7 + (-5) = **c)** -5 + 4 = **d)** 7 + (+5) =
   e) 4 + (-10) = **f)** -18 + (+10) = **g)** -5 + (-5) = **h)** 10 + (-7) =

2. Work out the following sums with integers:
   a) 10 + 5 + (-3) = **b)** 9 + (-3) + (-12) = **c)** 13 + 8 + 7 + (-1) = **d)** 6 + (-4) + (-3) + 8 =

3. Fill in the gap:
   a) 6 + ___ = 9 **b)** ___ + (-4) = 1 **c)** - 2 + ___ = -3 **d)** 3 + ___ = -4

4.2. Subtraction of integers

To **subtract** two integers we **add the first number to the opposite of the second number**.

For example:

5 - 2 = 5 + (-2) = 3 7 - (-3) = 7 + 3 = 10 -2 - (-7) = - 2 + 7 = 5 -8 - 3 = -8 + (-3) = -11

Exercises.

1. Work out:
   a) -2 - 8 = **b)** 6 - 7 = **c)** -19 + (-20) =
   d) -10 - (+4) = **e)** 3 - (-9) = **f)** 16 - (-2) =

2. Write down the missing numbers:
   a) -5 - (-6) = -5 + ___ = ___ **b)** 3 - (-8) = 3 + ___ = ___ **c)** -12 - ___ = -12 + 6 = ___

4.3. Brackets rules

TWO VERY IMPORTANT RULES YOU MUST REMEMBER!

Rule 1: If there is a **plus sign before a bracket**, **don't do anything:** remove it by writing its terms as they are.

For example:

+ (-7) = -7 + (+2) = +2 +(-5 + 6 – 7) = -5 + 6 - 7

Rule 2: If there is a minus sign before a bracket, **change positive signs** within it **to negative** and **negative signs to positive signs**.

- (-6) = + 6 - (+5) = -5 -(3 – 5 + 7) = -3 + 5 – 7 + 4 -6 - (+4) - (-7) + (-2) = - 6 – 4 + 7 - 2

4.4. Combined additions and subtractions

The best way to **solve combined additions and subtractions** of integers is:

- First, **remove brackets** (have a look at the rules shown in the previous section):
  For example:
  - 4 + (-3) - (-7) + (+4) = - 4 – 3 + 7 + 4
- After that, **add up all positive integers** and **all negative integers**. Look at the example:
  - 4 – 3 + 7 + 4 = -7 + 11
- Finally, subtract the smaller from the larger, and give the result the sign of the integer with the larger absolute value:
  -7 + 11 = + 4

Exercises

1. Work out the following combined additions:
   a) (-3) + (-7) – (+3) – (-3) + (-5) =
   b) -5 + (-3) – (-2) + (-5) – (-3) =

c) -(-5) + (-2) - (-3) + (-7) - (-2) =
d) 4 - (+4) - (-4) + (-3) - (-2) - (-7) =
e) -2 - 8 + (-3) - (-7) + (-2) - 4 =
f) 6 - 7 + (-3) - (-5) + 7 - (-8) =

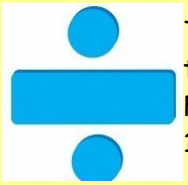
2. Fill in the missing number:
a) (-7) + _____ - 12 = 3 **b)** _____ -(-5) + 4 = 12 **c)** 8 + _____ - (-4) = -1
d) _____ - 3 + (-12) = -4 **e)** 17 + _____ - 5= 13 **f)** (-5) - _____ + (-7) = 2

4.2. Multiplication and division of integers

To **multiply** two or more integers we have to **multiply the signs** and then **the absolute value of the numbers**.

RULE OF THE SIGNS		
PRODUCT/DIVISION	+	-
+	Positive	Negative
-	Negative	Positive

For example:
2 · (-5) = -10 -3 · (-5) = 15 (-3) · 7 · (-1) · 2 = 42



To **divide** integers **the rules are the same than for the product**: divide the signs and then the absolute value of the numbers.
For example:
10 : (-5) = -2 (-15) : (-3) = 5 7 : (-1) = -7 -10 : (-2) = 5

Exercises.

1. Work out the following multiplications and divisions:
a) (-2) x (-7)= **b)** (-15) : (-3) = **c)** (-20) x (+1)=
d) (-25) : (+5) = **e)** (+3) x (-10)= **f)** (-10) : (-1) =

2. Fill in the gaps with a number:
a) (-2) x ___ = -22 **b)** (-15) : _____ = 15 **c)** ___ x (-5) = 30
d) _____ : (+5) = -10 **e)** (+6) x ___ = -18 **f)** (-70) : _____ = -7

4.3. Order of Operations

- Do all operations in **brackets** first.
- Work out the **exponents**.
- Then do **multiplications and divisions** in the order they appear (**from left to right**).
- Finally, do additions and subtractions in the order they appear.

Easy way to remember them!

Brackets **E**xponents **D**ivisions **M**ultiplications **A**dditions **S**ubtractions

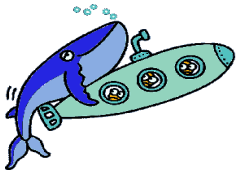
BEDMAS

Exercises. Find out the outcome:

a) 32 + (-12) : 6 =
b) (-8) · 9 – 15 · (-3) =
c) (-4) · 10 : 2 + 14 : (-7) =
d) 27 : (-3) · 2 – (-4) =
e) (-18) : 6 + 5 · (-10) =
f) 18 : 9 + 5 – (-15 · 3 + 12 · 4) =
g) (-6) · (4 – (-2)) + (-8 + (-3) · 2) =
h) (-35) : (5 + 2) + (-4) · 9 – (7 – 2 · 5) =
i) [(3 – 4) + (-2)] · 4 + 9 : (-3) · 6 =
j) [((-12) – (-3)) · 8] + 24: [((-2) + (-6)) : 2] =

EXTRA: Word problems

1. One day the temperature:
   a) goes up from –4°C to 12°C. What is the rise in temperature in °C?
   b) was 13 degrees and dropped 20 degrees. What is the temperature now?



2. A submarine ...
   a) was situated 450 m below sea level. If it ascends 175 m. What is its new position?
   b) was situated 370 m below sea level. If it descends 150 m. What is its new position?

3. Talking about mountains...
   a) Mt Everest, the highest elevation in Asia, is 8 848 m above sea level. The Dead Sea, the lowest elevation, is 409 m below sea level. What is the difference between these two elevations?
   b) The highest elevation in North America is Mt. McKinley, which is 20 320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level. What is the distance from the top of Mt. McKinley to the bottom of Death Valley?

4. You have:
   a) 270 euros in your bank account and you write a check for 125 euros. What is your balance?
   b) 300 euros in you bank account and you pay an expensive gift by credit card. If the gift costs 450 euros, what is your balance?