

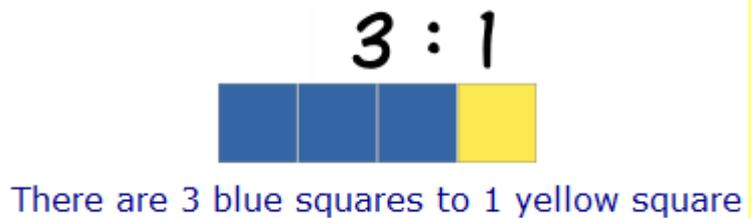
UNIT 8 VOCABULARY: PROPORTIONALITY

1.1. Ratio

A ratio says how much of one thing there is compared to another thing. If we want to compare two quantities, we can **divide** both numbers.

Ratios can be shown in different ways:

- Using the ":" to separate the values → 3 : 1
- Instead of the ":" symbol, you can use **the word "to"** → 3 to 1
- Or write it like a **fraction** → $\frac{3}{1}$



For example:

Janet has a bag with 4 pens, 3 sweets, 7 books, and 2 sandwiches.

- a) What is the ratio of books to pens?

Expressed as a fraction, the answer would be $\frac{7}{4}$.

Two other ways of writing the ratio are 7 to 4 (7 books to 4 pens), and 7:4.

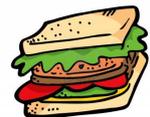


You can also write a ratio as a decimal number: $\frac{7}{4} = 1.75$

What's the meaning of this?

- b) What is the ratio of sweets to the total number of items in the bag?
There are 3 candies, and $4 + 3 + 7 + 2 = 16$ items total.

The answer can be expressed as $\frac{3}{16}$, 3 to 16, or 3:16.



Exercise. Write the ratio of each pair of quantities and explain its meaning:

1. In a class there are 18 girls and 12 boys.
 - a) Write the ratio of boys to girls.
 - b) Write the ratio of boys to the total number of students.
2. In a drink, 80 cm³ of pineapple juice is mixed with 20 cm³ of orange juice.
 - a) Write the ratio of pineapple juice to orange juice.
 - b) Write the ration of orange juice to pineapple juice.
3. 10 m of chain cost £6. Write the ratio of metres of chain to pounds.

1.2. Proportion

The height to width ratio of the Indian Flag is **2:3**, so for every 2 (inches, meters ...) of height there are 3 of width.



Therefore, if you make the flag 20 inches high, it must be 30 inches wide; and if you make the flag 40 cm high, it must be 60 cm wide (which is still in the ratio 2:3)

$$\frac{2}{3} = \frac{20}{30} = \frac{40}{60}$$

When two ratios are equal, the four terms **are in proportion** or **form a proportion**.

For example, $\frac{3}{7} = \frac{6}{14}$, so 3 : 7 and 6 : 14 form a proportion.



This is read "three is to seven as six is to fourteen".



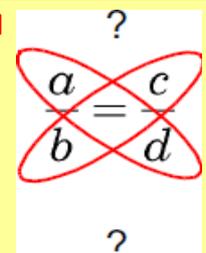
If we write a proportion as $\frac{a}{b} = \frac{c}{d}$

- b and c are called the **means**.
- a and d are called the **extremes**.
- The quotient of $a \div b$ or $c \div d$ is called **the proportionality constant**.

1.3. Fundamental property of proportions

A fundamental property of proportions is that **the product of the extremes is equal to the product of the means**.

$$a \cdot d = b \cdot c$$



This property is very useful when we know three of the numbers of a proportion and want to calculate the fourth.

If we have $\frac{a}{b} = \frac{x}{d}$, then $x = \frac{a \cdot d}{b}$

Exercises.

1. Write three different proportions with these ratios:

a) 3 : 2

b) $\frac{7}{5}$

c) 1.5 to 5

2. Find out the unknown number in these proportions

a) $\frac{2}{3} = \frac{5}{x}$

b) $\frac{5}{2.4} = \frac{x}{8}$

c) $\frac{x}{9} = \frac{12}{24}$

d) $\frac{2}{x} = \frac{9}{2}$

1.4. Fourth proportional

If you know three of the terms of a proportion, the unknown term is called the **fourth proportional**.

For example, to find the fourth proportional of $\frac{2}{7} = \frac{12}{x}$, we take cross-product:

$$x = \frac{7 \cdot 12}{2} = 42$$

1.5. Mean proportional

A **continued proportion** is a proportion that has **two equal means or two equal extremes**.

For example, $\frac{4}{6} = \frac{6}{9}$ is a continued proportion because its means are equal.

That equal term is called the **mean proportional**.

For example, to find the mean proportional of $\frac{9}{x} = \frac{x}{16}$, we take cross-product:

$$x^2 = 9 \cdot 16 = 144, \text{ so } x = \sqrt{144} = 12$$

Exercises.

1. Find x:

a) $\frac{4}{12} = \frac{12}{x}$

b) $\frac{18}{5} = \frac{7}{x}$

c) $\frac{20}{x} = \frac{x}{5}$

2. A car uses 22 litres of petrol to travel 176 miles. How far would the car travel using:

- a) 11 litres of petrol?
b) 33 litres of petrol?

3. If there are 4.55 litres in one gallon, convert:

- a) 20 gallons to litres.
b) 182 litres to gallons.

2.1. Direct proportions

We say that there is a **direct proportionality** between two magnitudes if **an increase on one magnitude causes a proportional increase on the other** and **a decrease on the first quantity causes a proportional decrease on the second**.

For example:

A recipe for pancakes uses 3 cups of flour and 2 cups of milk.

So the ratio of flour to milk is $\frac{3}{2}$



However, if you need to make pancakes for four of people you might need 4 times the quantity, so you multiply the numbers by 4:

$$\frac{3 \cdot 4}{2 \cdot 4} = \frac{12}{8}$$

In other words, 12 cups of flour and 8 cups of milk.

Number of people	1	2	4	10	20
Cups of flour	3	6	12	30	60
Cups of milk	2	4	8	20	40



The best way to recognize if two magnitudes are in a direct proportion is to see if when **we double one the other also doubles** and if we **half the first, the other also halves**.

2.2. Solving word problems: Rule of Three for direct proportionality

The **Rule of Three** is a very old method for solving proportions. It is very similar to the example above.

For example, we want to know how far a car gets in 7 hours, if we know that its speed is constant and that the car travels 90 miles in 3 hours.

90 miles	⇒	3 hours
x miles	⇒	7 hours



You can use the Rule of Three to calculate x directly: $x = \frac{90 \cdot 7}{3} = 210$ miles.



It is a good idea to write always the units so we can see that we are organising the quantities correctly.

Exercises.

- Richard earns £17.5 for working 7 hours. How much does he earn for working 9 hours?
- We pay 364€ for 7 nights in "Hotel los Llanos". How much do we pay for 3 nights? How much for 15 nights?
- We're cooking a cake for 6 people. The recipe says that we need 3 eggs, 150 g of flour and 50 g of sugar. Calculate how much of each ingredient we need to cook a cake for 9 people.
- We can paint 70 m² of wall with a 25 kg tin of paint. How many kg do we need to cover 53 m² of wall?
- My car uses 16 litres of petrol to travel 250 km.
 - How far can I travel with 55 litres?
 - How much petrol do I need to travel 180 km?
- In a drink, 53 mL of fruit juice are mixed with 250 mL of water. How many litres of water are there in 30 L of that drink?

3.1. Inverse proportions

We say that there is an **inverse proportionality** between two magnitudes if **an increase on one magnitude causes a proportional decrease on the other** and **a decrease on the first quantity causes a proportional increase on the second**.

For example, imagine that 18 men can do a job in 12 days. However, if you need to finish the job in just 6 days (**half**), you would need **twice men**, so you multiply men by two and divide days by two.



Men	18	36	9	3	180
Days	12	6	24	72	01/02/12

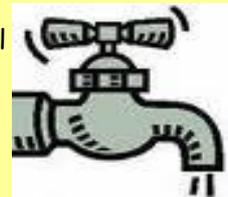
 The best way to recognize if two magnitudes are in a direct proportion is to see if when **we double one the other halves** and if we **half the first, the other doubles**.

3.2. Solving word problems: Rule of Three for inverse proportionality

You can also use the **Rule of Three** for solving inverse proportions.

For example, it takes 14 hours for a faucet with a flow of 18 litres per minute to fill a tank with water. How long will it take if its flow is reduced to 7 litres per minute?

14 hours \Rightarrow 18 litres per minute
 x hours \Rightarrow 7 litres per minute



The only difference with direct proportions is that you have to **invert one of the ratios (flip it upside down)**.

$\frac{x}{14} = \frac{18}{7}$ And finally we take cross-product: $x = \frac{14 \cdot 18}{7} = 36$ hours.

Exercises.

- Two pumps take 5 days to empty a pool. How long will 5 pumps take to empty the same pool?
- A truck that can carry 3 tons of sand. It needs 15 trips to carry a certain amount of sand. How many trips do you need to carry the same amount of sand with another truck that can carry 5 tons?
- It takes 12 hours for 3 bricklayers to build a wall. How long does it take for 5 bricklayers?
- James can write 8 pages with 25 lines in one hour. How many pages can he write if there are 20 lines on each page?
- If 2 eggs take 6 minutes to boil, how long do 5 eggs take?
- There are 13 winners in a lottery! Each one gets €23,000 . Imagine that there are 3 more winners with the same prize. How much does each one receive now?

4.3. Percentages

A **percentage** is as a fraction with denominator 100.
 For example:

• 75% really means $\frac{75}{100} = \frac{3}{4}$



- 100% is $\frac{100}{100}$, or exactly 1 (100% of any number is just the number, unchanged)
- 200% is $\frac{200}{100}$ or exactly 2 (200% of any number is twice the number)

A Percent can also be expressed **as a Decimal** or a **Fraction**. Look at the chart for more examples:

Percent	1 %	5 %	10 %	12.5 %	25 %	33.3 %	75 %	80 %	150 %
Decimal	0.01	0.05	0.1	0.125	0.25	0.333	0.75	0.8	1.5
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{3}{2}$

To **calculate the percentage of a quantity** we must multiply it by the percent and divide by 100, or just multiply by the equivalent decimal.

For example, to calculate the 35 % of 28, we can calculate:

- $28 \cdot 0.35 = 9.8$
- $28 \cdot \frac{35}{100} = 9.8$

Exercises.

- Convert these percentages to decimals:
75 % 1.74 % 80 % 3.7 % 0.6 % 9 %
- Convert these decimals to percentages:
0.5 0.05 0.35 0.1 0.075 0.94
- The population of a town is 652,000 people and 35% of them live in the centre district. How many of them live in this district?
- You have the possibility of choosing a prize:
 - Prize 1: 25% of 580€.
 - Prize 2: 30% of 450€.
 - Prize 3: 95% of 180€.

Which one do you choose? Explain your answer.

4.2. Common percentages

Some percentages are very common and it is useful to know them as fractions or decimals.

PERCENTAGE	FRACTION	DECIMAL	QUICK RULE
100 %	$\frac{100}{100} = 1$	1	Multiply by 1 100 % of 76 = $1 \cdot 76 = 76$
50 %	$\frac{50}{100} = \frac{1}{2}$	0.5	Divide by 2 50 % of 76 = $76/2 = 38$
25 %	$\frac{25}{100} = \frac{1}{4}$	0.25	Divide by 4 25 % of 76 = $76/4 = 19$
10 %	$\frac{10}{100} = \frac{1}{10}$	0.1	Divide by 10 10% of 76 = $76/10 = 7.6$
1 %	$\frac{1}{100}$	0.01	Divide by 100 1% of 76 = $76/100 = 0.76$

4.3. Word problems with percentages

Calculate a number increased or decreased in a percentage

Calculate the % of the quantity and then add or subtract the percentage to the original quantity.

For example:

The population of a town is 63,500. This year, it increases by 8%. What is the population now?

$$8\% \text{ of } 63,500 = 0.08 \cdot 63,500 = 5,080 \rightarrow 63,500 + 5,080 = 68,580 \text{ people}$$



PERCENTAGES AS DIRECT PROPORTIONS

Word problems involving percentages can also be solved using direct proportions.

For example:

I have bought a pair of jeans for €32. The VAT in UK is 20%. What was the price before VAT?

€ 32	⇒	120 %
€ x	⇒	100 %



$$x = \frac{32 \cdot 100}{120} = 26.67 \text{ So the price before VAT was € } 26.67$$

Exercises.

Shop Sale

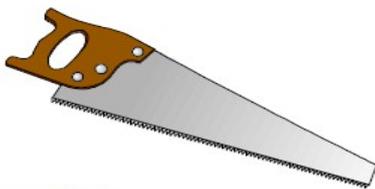
Hammer
£20



Compass
£12



Saw £16



Drill £60



1. The shopkeeper decides to have a sale. 50% off all items. What are the new prices of the items?
2. The shopkeeper then decides to have a 25% sale. What are the new prices of the items?
3. The shopkeeper then decides to have a 10% sale. What are the new prices of the items?
4. If you buy all four items in the 25% sale, how much do you spend?
5. The price of an electric oven is 560€ plus 17% VAT. What is the final price?
6. A friend diets and goes from 125 pounds to 110 pounds. What is her percentage weight loss?
7. Judy's salary increases from €1500 to €1590. What is the percent increase?
8. A test has 20 questions. If Peter gets 80% correct, how many questions does Peter miss?
9. 24 students in a class take a geography test. 18 students pass the test. What percent do not pass?