

**1. Algebraic expressions:**

It is an expression containing letters, numbers and mathematical signs ( +, -, ·, /, =)

They can express mathematic properties:

The commutative property of the multiplication:  $a \cdot b = b \cdot a$ , the distributive property:

$$a \cdot (b + c) = a \cdot b + c \cdot d, \text{ or a property of the powers: } a^b \cdot a^c = a^{b+c},$$

geometric formulas:

The area of a triangle:  $A = \frac{b \cdot h}{2}$ , the length of a circumference :  $L = 2\pi R$ , or the volume of a sphere:  $V = \frac{4}{3}\pi R^3$

Or physics equations:

The equation of movement when the velocity is constant  $v = \frac{s}{t}$ , or the second law of motion  $F = m \cdot a$

Algebraic expressions are useful to solve problems. How? Just translating the sentences to algebraic expressions:

Examples: Write the following sentences using algebraic expressions:

- a. Francisco is twice as much old as his son Pedro:  **$2x$**
- b. Marta is 5 years older than her husband:  **$x + 5$**
- c. Luisa's mother is three times older than Luisa:  **$3x$**
- d. Juan's sister is one year younger than Juan:  **$x - 1$**
- e. Two fifths of my income:  $\frac{2}{5}x$
- f. The cube of a number times the number plus eight:  **$x^3 \cdot (x + 8)$**
- g. The sum of two squared numbers:  **$a^2 + b^2$**
- h. The squared of two numbers sum:  **$(x + y)^2$**

## Exercises:

1. Let  $x$  represent a number. Write the following sentences using algebraic expressions:

- a. The number plus seven:
- b. One more than the number:
- c. Two less than the number:
- d. The number times four:
- e. The number times three plus five:
- f. The number times two minus eight:
- g. Half of the number:
- h. A third of the number plus nine:

2. Write in algebraic language:

- a. The product of two consecutive numbers:
- b. An even number:
- c. An odd number:
- d. The square root of two numbers difference:
- e. The perimeter of an equilateral triangle:
- f. The volume of a cube:
- g. The area of a rectangle whose length is twice its width:
- h. The area of a rectangular plot of land whose length is 5 metres less than triple its width:
- i. The side of a square whose perimeter is 10 metres long:

3. Fill in the table:

Algebraic expression	How it is read in Spanish	How it is read in English
$3x - 4$		
$\left(\frac{1}{2}x\right)^3$		
$\frac{x}{4} + 7x$		
$\frac{x+y}{6}$		

## 2. Numerical value

The **value** of an algebraic expression is its value when you replace the letters with numbers.

Example: The area of a triangle is  $A = \frac{b \cdot h}{2}$

Calculate:

Its value when  $b = 10\text{cm}$  and  $h = 7\text{cm}$  is  $\frac{10 \cdot 7}{2} = 35\text{cm}$

Its value when  $b = 6$  and  $h$  is twice its base:

4. Calculate the value of each expression if  $x = \frac{1}{2}$  and  $y = -2$ . You have to substitute  $x$  for  $\frac{1}{2}$  and  $y$  for  $-2$ :

a.  $5x + 4 = 5 \cdot \frac{1}{2} + 4 = \frac{5}{2} + 4 = \frac{5}{2} + \frac{8}{2} = \frac{13}{2}$

b.  $8 - 5y =$

c.  $\frac{1}{3}x - y^2 =$

d.  $xy - 5y + 7 =$

## 1. Monomials

A monomial is an algebraic expression containing just a product between a number (called coefficient) and one or more letters with natural exponents (literal part):

	Coefficient	Literal part
$-5x^2$	-5	$x^2$
$\frac{1}{6}xy$	$\frac{1}{6}$	$xy$
$-ab^2$	-1	$ab^2$
$15z^{-7}$	It is not a monomial because it has a not natural exponent.	

Two monomials are **like** when having the same literal part.

Reducing monomials:

Monomials can be added or subtracted only when they are like monomials:

Ex:  $10x + 8x = 18x$

$$5x^2 - x^2 = 4x^2$$

$$14a^3 + ab - 4a^3 + 9ab = 10a^3 + 10ab$$

An algebraic expression can be divided or multiplied by a number:

Ex:  $8 \cdot (2x^3 + 4x) = 16x^3 + 32x$

$$\frac{10x - 25}{5} = 2x - 5$$

5. Indicate if the several couples or monomials are like or not.

a.  $45a^2$  y  $3ab^2 \rightarrow$  Non like

b.  $-34x^4$  y  $3x^4$

c.  $5x^3y$  y  $6yx^3$

6. Complete the table:

Monomial	-3x	$5a^2$	$3ab^2$	$\frac{1}{2}x^2y^3$	$-34x^4$
Coefficient	-3		3		

Literal part	x		$ab^2$		
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7. Reduce the following expressions:

a.  $6x - 5x + x =$

b.  $2x + x + y - \frac{1}{3}y =$

c.  $3 \cdot (x - 4y) - 2x =$

d.  $(3a + b) - (a + 2b) =$

e.  $(-2) \cdot (-5b) =$

f.  $-\frac{1}{2} \cdot 8x^3 =$

g.  $5 \cdot (2x - y) - 3x =$

h.  $\frac{1}{3} \cdot (9x + 12) + 4 \cdot (4x - 1) =$

### **3. Equations and their components:**

An equation is an expression showing the equality of two algebraic expressions. This equality is true only when certain values are assigned to the letters.

For example, the equation:

$$x + 6 = 2x + 1$$

is true only when  $x = 5$ , since  $5 + 6 = 2 \cdot 5 + 1$

- On each side of the equal sign in an equation there is an **expression**
- The addends that comprise the expressions are called **terms**.
- The letters are called **unknowns**.
- The values for which the equation is true are called **solutions**.

#### 4. Solving first degree equations with one unknown

To solve a first degree equation with one unknown, you reduce the expressions and move the terms until the unknown is isolated.

Examples:

$6x - 4 = 5x + 3$ $6x - 5x = 3 + 4$ $x = 7$	$2x - 6 = 4 - 3x$ $2x + 3x = 4 + 6$ $5x = 10$ $x = 10/5$ $x = 2$	$7x - 4 = 2 - 2x$
$3(x - 5) = 4x - 2$ $3x - 15 = 4x - 2$ $3x - 4x = -2 + 15$ $-x = 13$ $x = -13$	$\frac{1}{2}(-8x + 10) = \frac{1}{3}$ $-4x + 5 = \frac{1}{3}$ $-4x = \frac{1}{3} - 5$ $-4x = \frac{1}{3} - \frac{15}{3} = \frac{-14}{3}$ $x = \frac{-14}{3} \cdot (-4) = \frac{56}{3}$	$\frac{2}{3}x + 20 = x$ $\frac{2}{3}x - x = -20$ $-\frac{1}{3}x = -20$ $x = 60$

#### 5. Solving problems with help of equations

Equations are useful tools that can help you to solve problems.

Example: Watch the steps taken.

In a pride of 13 lions, there are 3 more females than there are males. How many lions and how many lionesses are there?

<u>Step 1:</u> Identify the known and unknown numbers: Lions → x Lionesses → x+3 Total → 13	<u>Step 2:</u> Use an equal sign to state the relationship between the components of the problem.  Lions + lionesses = 13  $x + (x + 3) = 13$
<u>Step 3:</u> Solve the equation: $x + x + 3 = 13$ $2x = 13 - 3$ $2x = 10$ $x = 10/2$ $x = 5$	<u>Step 4:</u> Give a solution:  Lions → x=5 Lionesses → x + 3 = 5 + 3 = 8 Solution: There are 5 lions and 8 lionesses.

Word problems:

1. There are 31 people in a cafe. How many men and how many women are in the café if there are 5 more men than women?
2. If you add thirteen to two times a number, you obtain a result of 99. What is the number?
3. A bowl of cream costs eighty cents more than a pasty. Maria and Felisa bought one bowl of ice cream and two pasties for a total of €4.40. How much does a pasty cost? How much does a bowl of ice cream cost?
4. A box of figs weighs one kilo more than a box of strawberries. Together three boxes of strawberries and two boxes of figs weigh 12 kg. How much does each box weigh?

5. Calculate the dimensions of a rectangular plot of land, given that the plot's length is 20 metres more than its width and the fence that surrounds the plot is 240 metres long.