

MATHS

for 1st of E.S.O



I.E.S. LLANES

Index of contents:

Unit : Natural Numbers. Powers and roots.....	4
Unit : Multiples and factors,.....	6
Unit : Integer numbers.....	10
Unit : Fractions.....	12
Unit : Decimal numbers.....	14
Unit : Proportionality.....	17
Unit : Probability.....	21
Unit : Algebra.....	24
Unit: Geometry.....	27

NUMBERS

Cardinal numbers:

1	one	11	eleven	21	twenty-one	70	seventy
2	two	12	twelve	22	twenty-two	80	eighty
3	three	13	thirteen	23	twenty-three	90	ninety
4	four	14	fourteen	24	twenty-four	100	a/one hundred
5	five	15	fifteen	25	twenty-five	200	two hundreds
6	six	16	sixteen	26	twenty-six	1,000	a/one thousand
7	seven	17	seventeen	30	thirty	2,000	two thousands
8	eight	18	eighteen	40	forty	1,000,000	a/one million
9	nine	19	nineteen	50	fifty		
10	ten	20	twenty	60	sixty		

Ordinal numbers:

1st	first	11th	eleventh	21st	twenty-first
2nd	second	12th	twelfth	22nd	twenty-second
3rd	third	13th	thirteenth	23rd	twenty-third
4th	fourth	14th	fourteenth	24th	twenty-fourth
5th	fifth	15th	fifteenth	25th	twenty-fifth
6th	sixth	16th	sixteenth	30th	thirtieth
7th	seventh	17th	seventeenth	40th	fortieth
8th	eighth	18th	eighteenth	50th	fiftieth
9th	ninth	19th	nineteenth	100th	one hundredth
10th	tenth	20th	twentieth	1,000th	one thousandth

Use ordinal numbers

- to write fractions:

$\frac{5}{6}$	five sixths
$\frac{2}{3}$	two thirds
- in titles (names of kings and queens):

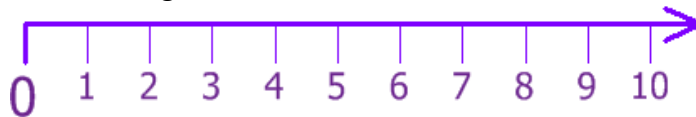
Charles II:	Charles the Second
Henry VIII:	Henry the Eighth

Number 0

Name	When we use it	Example
0 = zero	US English for the number In temperature	$3 - 3 = 0$ three minus three is zero $-10^{\circ} = 10$ degrees below zero
0 = Oh	After a decimal point In bus or room numbers In telephone numbers in years	7.03 "seven point oh three" Room 201 = Room two oh one Bus 107 = Bus one oh seven 9130472 = "Nine one three oh four seven two." 1802 = "Eighteen oh two."
0 = nought	before a decimal point	0.06 = "Nought point oh six."
0 = nil	in football	Chelsea 2 Manchester United 0 = "Chelsea two Manchester United nil."
0 = love	in tennis	20 - 0 = "Twenty - love."

Natural numbers: \mathbb{N}

Natural numbers or whole numbers are the numbers used for counting things.: 1, 2, 3,...(cardinal numbers) or for ordering things: 1st, 2nd, 3rd,(ordinal numbers)
It is a set with no ending: it has infinite elements that can be drawn in a ray:



Writing **big numbers**:

Using a place value chart can help you to write large numbers.

405 thousands = 405,000

45 thousands = 45,000

Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
M	HTh	TTh	Th	H	T	U
	4	0	5	0	0	0
		4	5	0	0	0

Operations:

Operation	symbol	word	example	
Addition or sum	+	Plus (and)	$3+7=10$	Three plus seven is/equals ten
Subtraction or difference	-	Minus (take away)	$15-6=9$	Fifteen minus six is/equals nine
Multiplication or product	x	Times or multiplied by	3×2	Three times two
Division or quotient	÷	Divided by	$50 : 5$	Fifty divided by five

In the sum $3+7 = 10$, 3 and 7 are addends, and 10 is the sum or total

In the difference $15-6 = 9$, 15 is the minuend and 6 the subtrahend; 9 is the difference

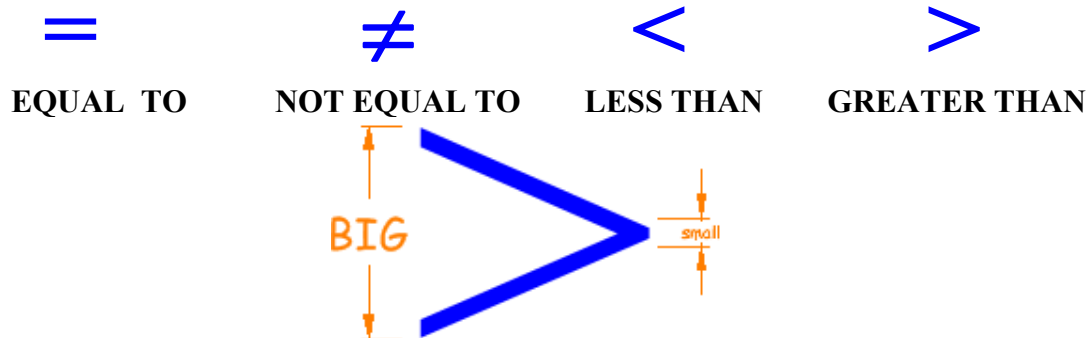
In the product 3×2 , 3 and 2 are factors

The ORDER of OPERATIONS



parentheses | powers & roots | multiplication & division from left to right |
addition & subtraction from left to right

Mathematical Symbols



Divisibility

The **multiples** of a number are what you get when you **multiply it by other numbers**:

The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, etc .

The multiples of 12 are 12, 24, 36, 48, 60, 72, etc

The **factors** or **divisors** of a number are the numbers that divide it exactly:

The factors of 12 are 1, 12, 2, 6, 3, and 4. (12 can be divided by all of them)

A **Prime number** has only two factors: itself and 1: 2,3,5,7.. are prime numbers

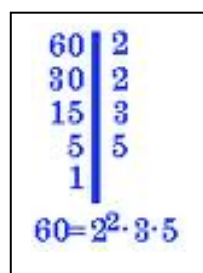
A **Composite** has other different factors : 4, 6, 8, 9, are composite numbers

The divisibility rules:

A number is divisible by	If:
2	The last digit is even (0,2,4,6,8)
3	The sum of the digits is divisible by 3
4	The last 2 digits are divisible by 4
5	The last digit is 0 or 5
6	The number is divisible by both 2 and 3
7	If you double the last digit and subtract it from the rest of the number and the answer is: <ul style="list-style-type: none"> • 0, or • divisible by 7 (Note: you can apply this rule to that answer again if you want)
8	The last three digits are divisible by 8
9	The sum of the digits is divisible by 9 (Note: you can apply this rule to that answer again if you want)
10	The number ends in 0
11	If you sum every second digit and then subtract all other digits and the answer is: <ul style="list-style-type: none"> • 0, or • divisible by 11

Factoring numbers:

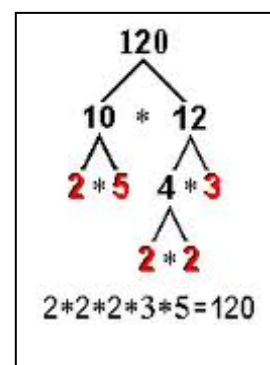
Dividing by its prime factors:



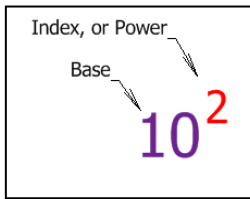
Lowest/least common multiple (LCD)= m.c.m.

Highest/gratest common divisor (HCD)= M.C.D.

Making a factor tree:



Powers and roots:



$$\sqrt{9} = 3$$

Use powers to multiply a number by itself several times

Names of powers:

3^2 is called 3 squared, 3 to the 2nd power or 3 to the power 2

4^3 is called 4 cubed, 4 to the 3rd power or 4 to the power 3

7^4 is called 7 to the 4th power or 7 to the power 4

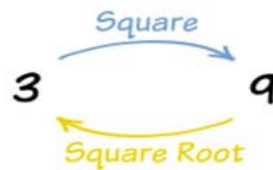
The opposite of squaring a number is called finding the square root.

The square root of 16 is 4 (because $4 \times 4 = 16$) $\sqrt{16} = 4$

The opposite of cubing a number is called finding the cube root

The cube root of 27 is 3 (because $3 \times 3 \times 3 = 27$) $\sqrt[3]{27} = 3$

Names or roots: Use ordinal numbers for roots with indices greater than 3, e.g.: 4th root, 5th root, etc



EXERCISES AND PROBLEMS

1. Write these numbers in words:

- 15 _____
- 50 _____
- 138 _____
- 9,324 _____

2. Use a place value table to write these big numbers with words and vice-versa:

- 158,432 _____
- 3,547,125 _____
- Twenty million, ten thousand, one hundred and twenty-eight

3. ¿What kind of number -even /odd- (par/impar) do you have if you add:

- even + even
- odd + odd
- even + odd?

4. Round to the nearest ten the following quantities:

- The distance from Glasgow to London is 418 miles.
- A theme park had 9,462 visitors last summer.

5. Find two numbers:

- with a product of 20 and a difference of 1
- with a sum of 7 and a product of 12
- with a sum of 8 and a quotient of 3

6. A car goes 560 km in 8 hours. What is its average speed?

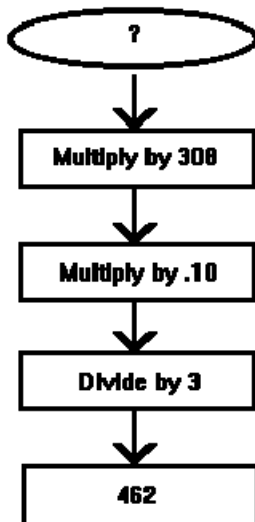
7. At the end of a concert five people have to pack away 465 chairs. How many chairs will each person have to pack away?

8. Find two consecutive numbers

- That add to 15.
- Whose product is 42.

9. Bill buys a second-hand television for \$156.00. He pays \$25.00 for repairs and sells it for \$197. What was his profit? (*to buy = comprar; to sell = vender; profit = beneficio*)

10. Find the first number:



11. Write the operations and find the results:

- Three more than five squared _____
- Six less than two cubed _____
- Twice the sum of three and seven _____
- Two plus eight divided by 2 minus 1 _____

12. Write in words:

- $3+21 > 12$ _____
- $25 \cdot 4 < 800$ _____
- $5^2-7 = 18$ _____

13. Complete the following sentences:

- A number is divisible by 2 if.....
- A number is a multiple of 3 if
- A number can be divided by 5 if.....

14. Justify your answers:

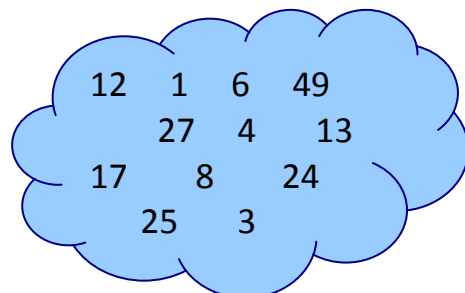
- Is **102** multiple of 3?
- Is **3435** divisible by 5?
- Is 6 a factor of **606**?

15. James goes running twice a week and Martha three times a week. If they meet on the 5th of January, when will they meet next time?

16. John has a number of pencils between **76** and **108**. He can group them in groups of 5 and in groups of 7. How many pencils has he got?

17. Look at the numbers in the cloud and write down the numbers that are;

- Factors of 24
- Prime numbers
- Square numbers
- Multiples of 6



Negative numbers. Integer numbers

A **negative number** is any number whose value is less than zero. We write a minus in front of negative numbers. Negative numbers are used for:

- Temperatures (below zero)
- Bank balances which are overdrawn
- Golf
- Uderground parkings
- Walking backward and forward (if the steps are the same length)

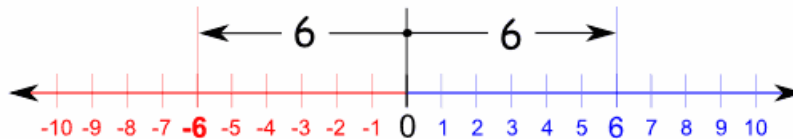


Both negative and positive numbers are integers and draw them on a Number line. The set of all the integer numbers is named **Z** and it doesn't have a beginning or an ending: it has infinite





Numbers on the right are bigger than numbers on the left: *-8 is smaller than -5*

The **absolute value** of a number is the distance from this number to 0:















The absolute value of an integer is always a positive number or 0: $|-6| = 6$; $|0| = 0$

Simplify signs before adding/subtracting integer numbers

	Rule	Example
	Two like signs become a positive	$+(+) \text{ or } 3+(+2) = 3+2 = 5$ $-(-) \text{ or } 6-(-3) = 6+3 = 9$
	Two unlike signs become a negative	$+(-) \text{ or } 7+(-2) = 7-2 = 5$ $-(+) \text{ or } 8-(+2) = 8-2 = 6$

Rules of signs when multiplying and dividing integers

 x 	two positives you get a positive:	 $3 \cdot 2 = 6$
 x 	a positive and a negative you get a negative:	 $(-3) \cdot 2 = -6$
 x 	a negative and a positive you get a negative:	 $3 \cdot (-2) = -6$
 x 	two negatives you get a positive:	 $(-3) \cdot (-2) = 6$

EXERCISES AND PROBLEMS

- Order these temperatures on the thermometer: 25°C , -31°C , -3°C , 1°C
- Work out what temperature is:
 - 3°C warmer than -1°C
 - 6°C colder than 3°C
 - 8°C warmer than -7°C
 - 6°C colder than 2°C
- What number are you on if you:
 - Start at **2** and take away 6
 - Start at **-7** and add on **3**
 - Start at **5**, take away 8 and add on **1**
 - Start at **-3**, add on 2 and take away **3**
- A proton has a charge of positive one. An electron has a charge of negative one. Find the total charge of an ion with 14 protons and 19 electrons.
- Work out in your head:

a) $3 - 2 - 7$	b) $-30 \div (-6)$	c) $-35 \div (+7)$
d) $(-2)^3$	e) $(-10)^4$	f) -10^4
g) $5 - 9 + 3 - 2$	h) $-(-(-(-7)))$	i) $-20 \div (-10) + (-10) \div 5$
j) (-1^4)	k) $(-1)^4$	l) $2 - (-(-2))$
- Copy and answer these questions in your notebook:
 - What are the integer numbers?
 - Which letter is used to name the set of all the integers?
 - Write two integer numbers that are natural numbers
 - Write two integer numbers that are not natural numbers
 - What sign is the opposite of a negative number?
 - What sign is $-|-3|$?

Write down all the new vocabulary of this lesson

:

Fractions: a fraction is a quotient of two integer numbers:

$\frac{1}{6}$ ← The top number is called the numerator.
← The bottom number is called the denominator.

Names of fractions:

To name fractions use ordinal numbers, the same as in Spanish:

1/8 an/one eighth
1/10 a/one tenth
1/16 a/one sixteenth
2/3 two thirds
3/5 three fifths

When the denominator is too large, you can name the fraction this way:

25/74 twenty-five *over* seventy four

Exceptions for this rule:

1/2 a/one half 1/4 a/one quarter
5/2 five halves.....3/4 three quarters

Proper fraction: has a numerator smaller than its denominator, eg.: **3/7**

Improper fraction: has a numerator bigger than its denominator, eg.: **11/5**

Mixed number: is a whole number together with a proper fraction $2\frac{5}{7}$

Equivalent fractions are fractions with the same value. You can get equivalent fractions by *simplifying* or amplifying (dividing or multiplying numerator and denominator by the same number)

Simplest form of a fraction (or fraction in lowest terms): when a fraction cannot be reduced it is in its simplest form. E.g.: 5/3, 12/25,.....

Adding and subtracting fractions: write them using a common denominator (LCD)

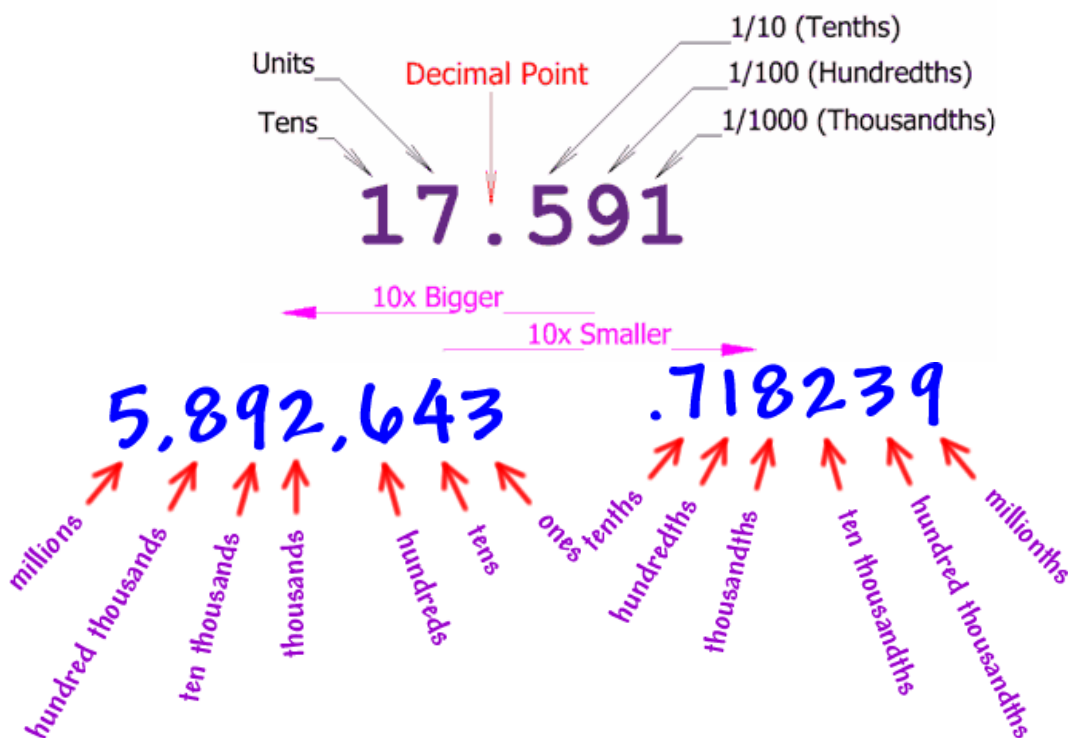
Multiplying and dividing fractions. Cancel factors before if it is possible.

Raising fractions to a power: raise to the power both, the numerator and the denominator.

EXERCISES AND PROBLEMS

- Find the simplest fraction for each one of these situations:
 - 80 of 100 people will pass next exam.
 - One day has 24 hours; Marta spends: 8 hours sleeping, 3 hours eating, 6 hours at school
- Write these numbers as fractions: $3=$, $-7=$, $11=$ $-1=$
- Which of the following fractions are proper fractions? For those that are improper, write them as a mixed number. a) $\frac{7}{4}$, b) $\frac{3}{5}$ c) $\frac{11}{9}$
- Factorise the terms of each fraction and cancel the repeated factors to simplify:
$$\frac{36}{64} \quad \frac{45}{54} \quad \frac{27}{18} \quad \frac{72}{63}$$
- There are 24 pupils in a class. $\frac{1}{3}$ of them like football, $\frac{3}{4}$ go swimming twice a week and $\frac{5}{6}$ love Maths. How many children like football? How many go swimming? Love Maths?
- How many minutes is $\frac{3}{5}$ of an hour?
- There are 120 pages in a book. If I've read $\frac{3}{8}$, what page am I on?
- If I can run 1 km in 12 minutes, how many metres could I run in $\frac{1}{4}$ of the time?
- Charlotte's take home pay is £930. She gives her mother $\frac{1}{3}$ of this and spends $\frac{1}{5}$ of the £930 on going out. What fraction of the total is left? Give your answer as fraction in its lowest terms.
- Answer each question, choose the correct answer or fill in the blank as appropriate.
 - You have to find a common denominator in which of these operations: addition, subtraction, multiplication, or division?
 - Given $\frac{9}{12} = \frac{\quad}{20}$ what is the numerator of the second fraction? How did you find it?
 - Explain the error: $\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$
 - Could you tell a friend how to change an improper fraction into a mixed number?

Decimal numbers: to represent numbers less than the unit, we use decimal numbers and the decimal point to separate them:



How do we **name or read** a decimal?

Ignore the decimal point and read the number as a whole number; then say the **decimal unit** where the last digit falls.

Example 1.

.038 "38 thousandths"

Example 2.

.002135 "2,135 millionths"

Example 3.

14.29 "14 and 29 ten-thousandths."

Rounding decimals: Rounding means reducing the digits in a number while trying to keep its value similar. *The result is less accurate, but easier to use.*

How to Round Numbers:

- Decide which is the last digit to **keep**
- Leave it the same if the **next digit** is less than 5 (this is called *rounding down*)
- But increase it by 1 if the next digit is 5 or more (this is called *rounding up*)

Examples of rounding:

Number	Rounded to the tenths	hundredths	thousandths	ten thousandths
3.141592	3.1	3.14	3.142	3.1416
1.263504	1.3	1.26	1.264	1.2635

Metric system and imperial units:

By the eighteenth century, dozens of different units of measurement were commonly used throughout the world. Length, for example, could be measured in feet, inches, miles, spans, cubits, hands, furlongs, palms, rods, chains, leagues, and more. The lack of common standards led to a lot of confusion and significant inefficiencies in trade between countries. At the end of the century, the French government sought to alleviate this problem by devising a system of measurement that could be used throughout the world. In 1790, the French National Assembly commissioned the Academy of Science to design a simple decimal-based system of units; the system they devised is known as the metric system. In 1960, the metric system was officially named the Système International d'Unités (or SI for short) and is now used in nearly every country in the world except the United States. The metric system is almost always used in scientific measurement.

In the Metric system, each type of measurement has a base unit to which the following prefixes are added to indicate multiples or submultiples of ten:

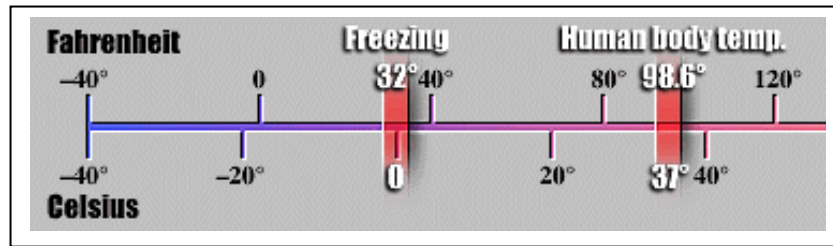
MULTIPLICATION FACTOR	PREFIX	SYMBOL
1,000,000,000 = 10^9	giga	G
1,000,000 = 10^6	mega	M
1,000 = 10^3	kilo	k
100 = 10^2	hecto	h
10 = 10^1	deka	D
1 = 1		
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000001 = 10^{-6}	micro	μ
0.000000001 = 10^{-9}	nano	n

The Imperial System was developed over hundreds of years in the UK. When the French developed the Metric System (metres) in 1670, it soon spread through Europe, even to England itself in 1963. But the USA and a few other countries still prefer feet and inches. Here you are some imperial units:

Length		
Imperial		Metric
1 inch [in]		2.54 cm
1 foot [ft]	12 in	0.3048 m
1 yard [yd]	3 ft	0.9144 m
1 mile [mi]	760 yd	1.6093 km
1 nautical mile	2025 yd	1.853 km

Mass		
1 ounce [oz]		28.35 g
1 pound [lb]	16 oz	0.4536 kg
1 stone	14 lb	6.3503 kg
1 long ton (UK)		1.0160 t

Temperature



The first mercury thermometer and temperature scale were introduced in Holland in 1714 by Gabriel Daniel Fahrenheit. Fahrenheit established three fixed points on his thermometer. Zero degrees was the temperature of an ice, water, and salt mixture, which was about the coldest temperature that could be reproduced in a laboratory of the time.

In 1743, the Swedish astronomer Anders Celsius devised the aptly-named *centigrade* scale that places exactly 100 degrees between the two reference points defined by the freezing- and boiling points of water.

Ratio and proportion. Percentages.

- **Ratio** is a way of comparing amounts of something. It shows how much bigger one thing is than another. For example: Use 3 parts blue paint to 1 part white.



- **Proportion** is an equation with a ratio on each side. It is a statement that two ratios are equal. Parts of a proportion:

$$\begin{array}{c} \text{a} \\ \text{b} \end{array} = \begin{array}{c} \text{c} \\ \text{d} \end{array} \text{ means extremes} \quad \begin{array}{c} \text{a} : \text{b} = \text{c} : \text{d} \\ \text{means} \\ \text{extremes} \end{array}$$

- **Direct proportion:** If two quantities are directly proportional then their quotient is an invariant (it does not change, it is constant). Example in the table:

A	2	3	4	5
B	6	9	12	15

- **Inverse proportion:** If two quantities are inversely proportional, then their product is an invariant (it does not change, it is constant). Example in the table:

C	12	6	3	2
D	2	4	8	12

- **No proportion:** How can you tell when a relationship between two numbers is a direct proportion, an inverse proportion, or not a proportion at all? If dividing values in corresponding columns of a table always gives the same number, there is a direct proportion. If multiplying values in the corresponding columns of a table always gives the same number, there is a direct proportion. Otherwise, there is no proportion.

E	1	2	3	4
F	2	3	4	5

- **Percentages:** To find 9% of 300: $9 \cdot 300 \div 100 = 27$
- **Map scales:** A scale of 1:50,000 means 1 unit on the map represents 50,000 of the same units on the ground.

EXERCISES AND PROBLEMS

Fraction (words)	Fraction (figures)	Decimal (figures)	Decimal (words)
One Unit and Two Tenths	1 2/10	1.2	One Point Two
Two Units and Four Tenths	2 4/10	2.4	Two Point Four
Three Units and Five Tenths	3 5/10	3.5	Three Point Five
Five Units and Six Tenths	5 6/10	5.6	Five Point Six
Five Units and Nine Tenths	5 9/10	5.9	Five Point Nine
Six Units and Seven Tenths	6 7/10	6.7	Six Point Seven
Seven Units and One Tenth	7 1/10	7.1	Seven Point One
Eight Units	8	8	Eight
Eight Units and Six Tenths	8 6/10	8.6	Eight Point Six

1. Answer these questions:

- I have one unit and seven tenths. Write this number in decimals.
- What is nine units and five tenths in decimals?
- Why do we write the decimal point in the number 6.4?
- How many tens, units and tenths are there in 75.6?

2. Read the words and write the number down in figures. Don't forget the decimal point!

- a. 7 units and 8 tenths _____
- b. 9 units and 6 tenths _____
- c. 5 units and 2 hundredths _____
- d. 8 tens, 3 units and 7 tenths _____
- e. 15 hundreds, 4 units and 5 tenths _____
- f. 75 millionths _____
- g. 87 units and 16 thousandths _____

3. How many numbers can you write between 0.23 and 0.24 ? Write five of these numbers.

4. What is the difference between 150.7 and 150.70?

5. Write the following measures in metric units:

- a) 5 feet, b) 27 inches c) 750 miles d) 12 pounds

6. How tall are you? How tall are you for British people?
7. Who is taller, Jane who is 5.7 feet or Julia who is 1.72 m.?
8. Who is the winner of the race: Martha: 2h, 27 min, 15 s, James: 187.3 min or Anne: 2h, 1350 s
9. Look at the picture and answer: The ratio frogs/birds is: _____



Complete the table keeping in mind this ratio:

Frogs	15			1	
Birds			12		
Total		27			81



10. *A box of sweets contains chocolates and toffees.*
 - a. *If one in every five sweets is a toffee, what proportion are chocolates?*
 - b. *There are 10 toffees. How many chocolates?*
 - c. *If there are 20 sweets altogether how many are chocolates?*
 - d. *Mary only likes toffees and Tim loves chocolate. If Mary eats all the toffees from a box containing 40 sweets. How many chocolates are left for Tim?*

11. A bag of M & M sweets contain just yellow and orange sweets. For every 2 yellow sweets, there are 6 orange sweets. Complete this table:

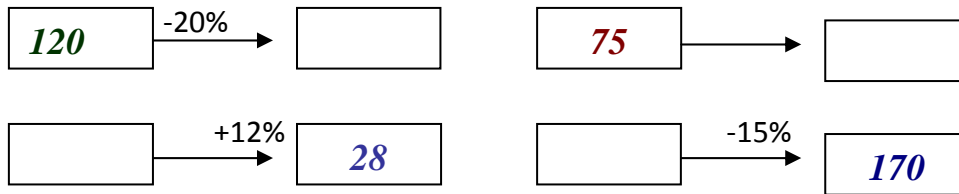
Yellow		4	6		
Orange	6	12			
Sweets			24		40

Now answer these questions.

- a. *What is the ratio of orange to yellow sweets?*
 - b. *If you have 8 yellow sweets, how many orange sweets will you have?*
 - c. *There are 32 sweets in the medium sized bag. How many will be yellow?*
 - d. *In the super fat size there are 40 sweets. What proportion are orange?*
12. From a school survey (*encuesta*) we know that for every 5 girls there are 3 boys.
 - a. *What is the ratio of girls to boys?*
 - b. *In a class of 27 children, how many are girls?*
 - c. *In 54 children, how many are boys?*
 13. In Great Britain you have to pay with pounds (£). You can get £20 for 30 €. Fill in the gaps:
 - a. 6 € = £ _____

- b. 9 € = £ _____
 c. £ 12 = _____ €
 d. £ 15 = _____ €

14. Fill in the gaps:



15. Which is the biggest: 20% of 1500 or 15% of 2000?
16. Three years ago, Miguel was 164 cm tall. How tall is he nowadays if he grows 5 % every year
17. You and your friends want to buy a present that costs 60 €. Fill in the gaps:
- a. If you are 5 friends, you'll payeach.
 - b. If you are 8 friends, you'll pay.....each.
 - c. If each one has to pay 10 €, you'll be.....friends.
 - d. If each one has to pay 15 €, you'll befriends.
18. Nine students have to do a composition of about 180 pages for homework .
- a. How many pages does each one have to write?
 - b. If there are 3 more students, how many pages will each one have to write?
19. William goes to school by bike. There are 400 metres between William's house and his school.
- a. If he gets to school in 16 minutes, what is his speed?
 - b. Last Tuesday he got to school in 20 minutes. What was his speed?
20. Six painters paint a wall in eight days. How many painters do you need to paint the same wall in only six days?

<http://www.skillsworkshop.org/n/fractions12.pdf>

<http://www.teachingideas.co.uk/maths/decnolines.htm>

<http://www.superteacherworksheets.com/decimals.html>

PROBABILITY

In the real world, events can not be predicted with 100% certainty. The best we can do is say how likely they are to happen, using the concept of probability.

Tossing a Coin

When a coin is tossed, there are two possible outcomes: heads (H) and tails (T).



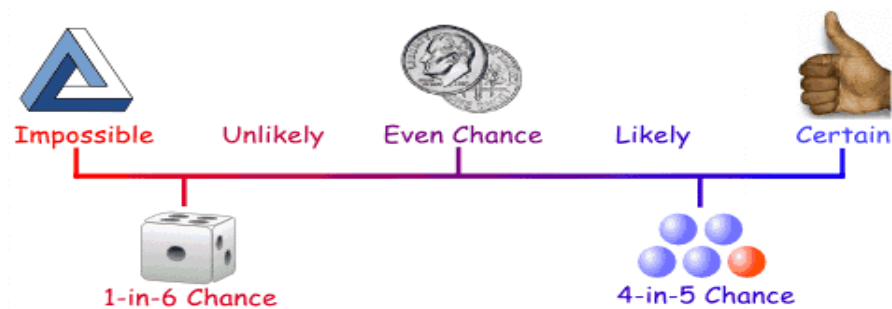
Throwing or rolling Dice

When a single die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6.

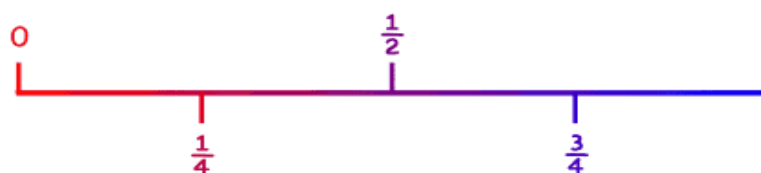


Probability is the chance that something will happen - how likely it is that some event will happen.

Sometimes you can measure a probability with a number: "10% chance of rain", or you can use words such as impossible, unlikely, possible, even chance, likely and certain



As well as words we can use fractions or decimals to show the probability of something happening. Impossible is **zero** and certain is **one**. A fraction probability line is shown below.



Remember!! **the probability of an event will not be more than 1.**

This is because 1 is certain that something will happen.

And **the probability of an event will not be less than 0.**

This is because 0 is impossible (sure that something will not happen).

In general:

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Example: there are 5 marbles in a bag: 3 are red, and 2 are blue. What is the probability that a blue marble will be picked?

Number of ways it can happen: 2 (there are 2 blues)

Total number of outcomes: 5 (there are 5 marbles in total)

So the probability is $2/5 = 0.4$

WARM UP!

- Write the correct number for the vocabulary word that matches each meaning.

1. certain	_____	The chance that an event will happen.
2. more likely	_____	Not capable of happening.
3. outcomes	_____	To be sure.
4. less likely	_____	Greater chance that an event will happen.
5. probability	_____	Smaller chance that an event will happen
6. impossible	_____	Outcomes that have the same chance of occurring.
7. equally likely	_____	Possible results of a probability experiment
8. chance	_____	A possibility or likelihood.

- Which of the following has a probability closest to 0.5?
 - A** The sun will rise tomorrow.
 - B** It will rain tomorrow.
 - C** You will see a dog with only three legs when you leave the room.
 - D** A fair die will come up with a score of 6 four times in a row.
 - E** There will be a plane crash somewhere in the world within the next five minutes.
- Which of the following is an experiment?
 - Tossing a coin

- Rolling a single 6-sided die
- Choosing a marble from a jar
- Which of the following experiments does NOT have equally likely outcomes?
 - Choose a number at random from 1 to 7.
 - Toss a coin.
 - Choose a letter at random from the word SCHOOL
- Write the sample space when two coins are tossed.

PROBLEMS

1. A standard deck of cards contains 52 cards. There are 4 suits: hearts, diamonds, clubs and spades. Each suit contains 13 cards. You are asked to select a card from the deck without looking. What is the probability of drawing a ten?



2. A bottle contains 8 marbles: 3 are red and 5 are blue. You are to take a marble from the bottle without looking. What is the probability that you will pick a red marble?



3. Our alphabet contains 26 letters. Five of these letters are vowels (A, E, I, O, U). The remaining 21 letters are called consonants. Each letter of the alphabet is written on a card and you are asked to choose a card. What is the probability of choosing a consonant?
4. A six-sided die is rolled. What is the probability of rolling a 6?



5. A six-sided die is rolled. The sides show dots that range from 1 dot to 6 dots. What is the probability that when rolled, the side with 3 dots will appear on the top of the die?
6. A coin is tossed. It is a fair, unbiased coin. What is the sample space? What is the probability that a head will appear?
7. A class contains 18 boys and 12 girls. The teacher calls on students at random to recite. What is the probability that the first student called upon to recite is: a boy? a girl?



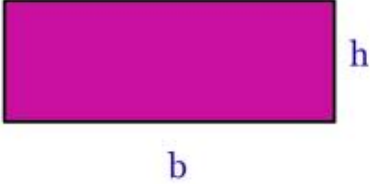
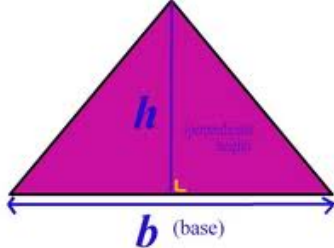
8. A letter is chosen at random from a given word. Find the probability that the letter is a vowel if the word is: a. ALGEBRA b. PROBABILITY
A pair of dice is rolled. What is the probability of getting a sum of

ALGEBRA



Algebraic expressions are expressions using letters to represent numbers

We use algebraic expressions in formulae, to solve problems, and in many different situations..

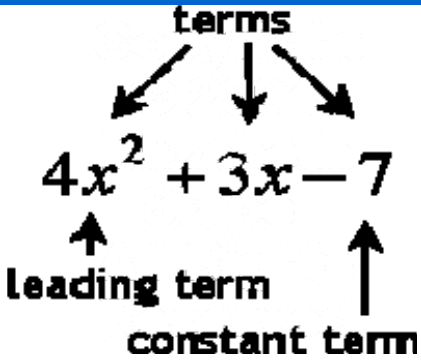
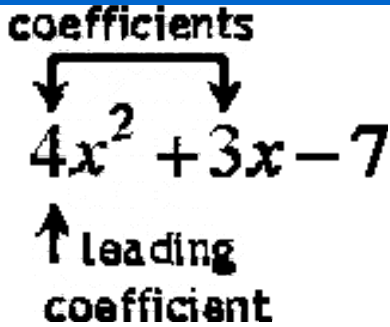
 <p style="margin-top: 20px;">Area = $b \times h$</p>	$\frac{4}{1} = \frac{12}{x}$  <p style="margin-top: 10px;">Area = $\frac{bh}{2}$</p>
---	---

Algebraic expression contains numbers and variables related with mathematical operations. The simplest expressions are polynomials.

Polynomials consist of terms which are made of coefficients and variables

Name	Number of Terms	Example
<i>Monomial</i>	1 (<i>monoimpliesone</i>)	$4x^2$
<i>Binomial</i>	2 (<i>biimpliestwo</i>)	$4x^2 + 3x$
<i>Trinomial</i>	3 (<i>triimpliesthree</i>)	$4x^2 + 3x + 5$

Like terms have the same letters and the same indices: $2x^2y$ and $-3x^2y$

<p>terms</p> 	<p>coefficients</p> 
---	---

Learn to solve word problems

1. Read the problema carefully.
2. Identify key variables (unknowns).
3. Eliminate unneeded variables.
4. Use the text of the problem to write equations.
5. Solve the equation.
6. Find the remaining variables.

EXERCISES:

1.-Write each sentence as an algebraic expression or as an equation:

- Eleven less than three times a number
- Fifty divided by two twice a number.
- Twelve less than a number is thirty
- Twice a number, decreased by nineteen
- A number increased by seventeen is twenty-two.
- The product of two consecutive integer numbers.
- The sum of three consecutiveeven numbers.

2.- Evaluate the expressions in this table:

Expression	$3x-1$	$5-2x$	x^2-4
$x = 2$			
$x = -1$			
$x = -3$			
$x = 1/2$			

3.- Write one equation for each of the following and then solve the equation:.

- *Peter opened a bank account with a certain amount of money. If he adds \$5.50 every week for one year, he will have \$300. What amount of money did he originally start with?*
- *Thomas went shopping and bought 15 pencils. He paid with a \$5 note and he got change of \$1.25 . How much did each pencil cost?*
- *The following formula converts temperatures from degrees Celsius to degrees Fahrenheit: $F = \frac{9C}{5} + 32$.*

How many degrees Celsius would be if it was 100° Fahrenheit?

4.- Use the table below to solve this problem: John is twice as old as his friend Peter. Peter is 5 years older than Alice. In 5 years, John will be three times as old as Alice. Howoldis Peter now?

	<i>Age now</i>	<i>age in 5 years</i>
Peter		
Alice		
John		

5.- The sum of the least and greatest of 3 consecutive integers is 60. What are the values of the 3 integers?

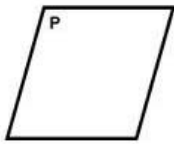
6.- The lengths of the sides of a triangle are consecutive odd numbers. What is the length of the longest side if the perimeter is 45?

7.- Karin’s mom runs a dairy farm. Last year Betty the cow gave 375 gallons less than twice the amount from Bessie the cow. Together, Betty and Bessie produced 1464 gallons of milk. How many gallons did each cow give?

Key words in problem solving

Addition	increased by more than combined, together total of sum added to
Subtraction	decreased by minus, less difference between/of less than, fewer than
Multiplication	of times, multiplied by product of increased/decreased by a factor of (this type can involve both addition or subtraction <i>and</i> multiplication!)
Division	per, a out of ratio of, quotient of percent (divide by 100)
Equals	is, are, was, were, will be gives, yields sold for

PLANE GEOMETRY (REVISION)



PLANE: a plane is a flat surface



A segment is a part of a line



A ray has an end point



A line extends infinitely far in two opposite directions

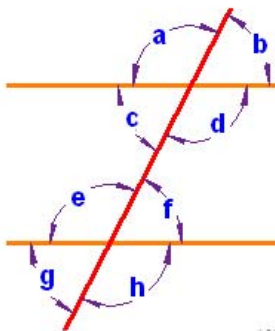
TYPES OF ANGLES:



Type of Angle : Description :

Acute Angle	an angle that is less than 90°
Right Angle	an angle that is 90° exactly
Obtuse Angle	an angle that is greater than 90° but less than 180°
Straight Angle	an angle that is 180° exactly
Reflex Angle	an angle that is greater than 180°

PARALLEL LINES AND PAIRS OF ANGLES

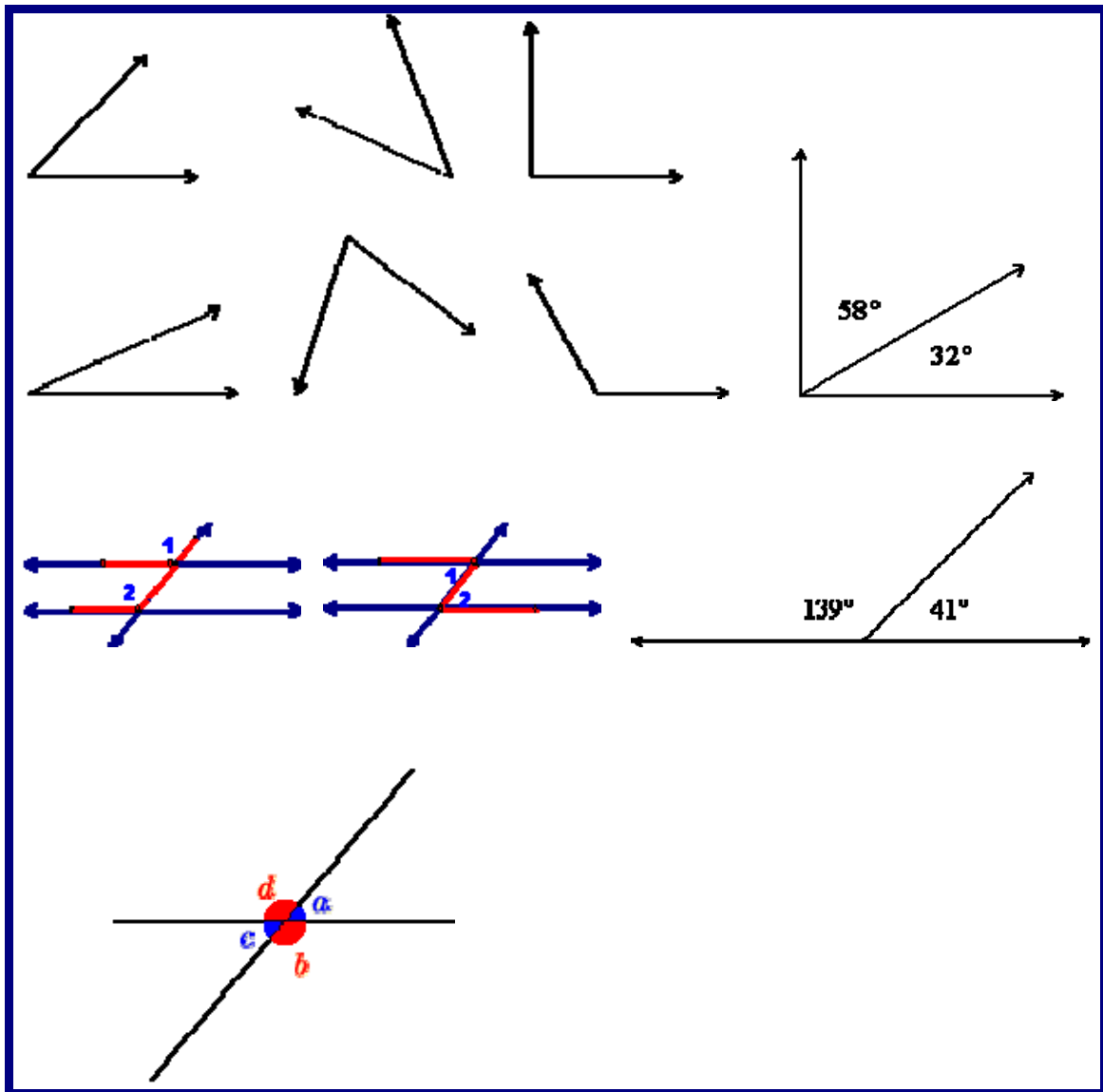


Corresponding Angles are equal: $a = e$
 Alternate Interior Angles are equal: $c = f$
 Alternate Exterior Angles are equal: $b = g$
 Consecutive Interior Angles add up to 180° : $d + f = 180^\circ$

1. Complete the following sentences:

- Two angles which add up to 180° are _____ angles.
- Two angles which add up to 90° are _____ angles.
- An _____ angle is less than 90° .
- An _____ angle is bigger than 90° .
- A _____ angle is bigger than 180° .
- _____ lines are lines that never meet.
- _____ lines are lines that meet on one point.
- One point on a line divides it into two _____ .

2. Classify these angles or pairs of angles:



3. Calculate the measurement of the complementary angle and the supplementary angle of these: 92° , 35° , 20° .
4. Order these words to make sentences:

*meet parallel
never lines*

*Than is
angle less
acute 90° An*

*add angles to
supplementary up
 180°*

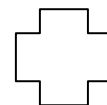
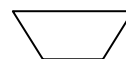
5. Justify your answers:

- a. Which capital letters have vertical line symmetry?
- b. Which capital letters have horizontal line symmetry?
- c. Which capital letters have BOTH vertical and horizontal symmetry?

6. Which of the following lettered items possesses point symmetry?

- a. the letter D
- b. a square
- c. the letter S
- d. the word **mow**
- e. the letter B
- f. the word DAD
- g. the letter Z

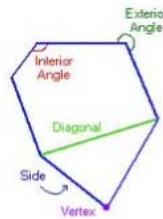
7. Draw the line(s) of symmetry of these figures:



POLYGONS

A polygon is an enclosed area bounded by at least three sides

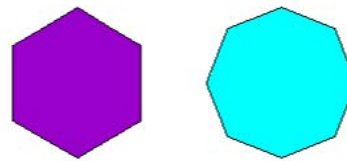
Elements in a polygon



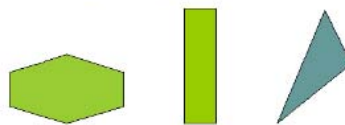
Regular Polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same. The sum of the angles of a polygon with n sides, where n is 3 or more, is $180^\circ \times (n - 2)$ degrees.

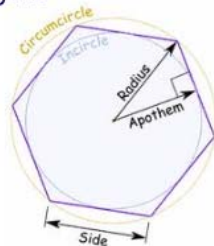
The following are examples of regular polygons:



The following are not examples of regular polygons:



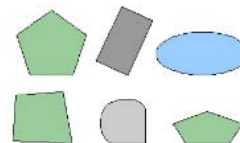
Elements of regular polygons:



Convex

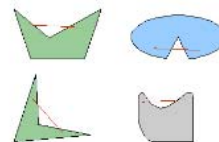
A polygon is convex if every line segment drawn between any two points inside the figure lies entirely inside the figure. A figure that is not convex is called a concave figure.

The following figures are convex.



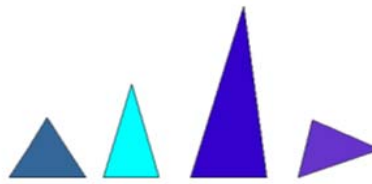
Concave

The following figures are concave. Note the red line segment drawn between two points inside the figure that also passes outside of the figure.



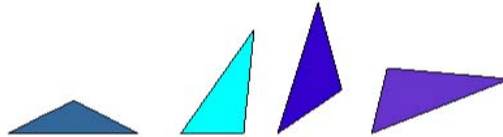
Acute Triangle

A triangle having three acute angles.



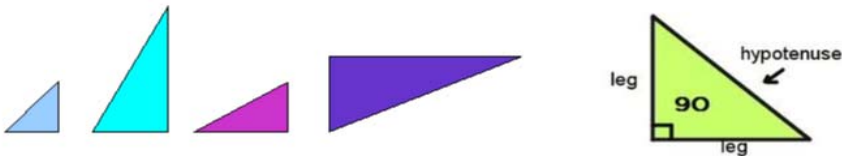
Obtuse Triangle

A triangle having an obtuse angle. One of the angles of the triangle measures more than 90 degrees.



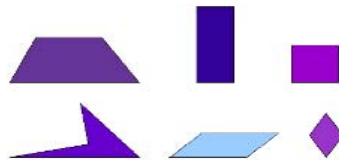
Right Triangle

A triangle having a right angle. One of the angles of the triangle measures 90 degrees. The side opposite the right angle is called the hypotenuse. The two sides that form the right angle are called the legs.



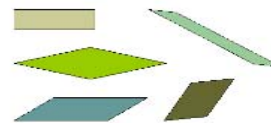
Quadrilateral

A four-sided polygon. The sum of the angles of a quadrilateral is 360 degrees.



Kinds of quadrilaterals:

Parallelogram: two pairs of parallel sides and two pairs of equal angles.



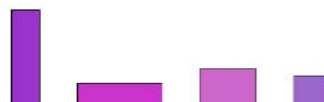
Trapezium: one pair of parallel sides (bases).

Tapezoid: no parallel sides.

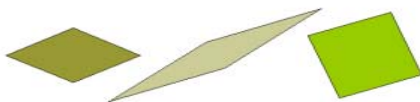
Squares (4 equal sides and angles)



Rectangles (4 equal angles and 2 pairs of equal sides)



Rhombuses (4 equal sides and 2 pairs of equal angles)



Pentagon

A five-sided polygon. The sum of the angles of a pentagon is 540 degrees.

A regular pentagon:



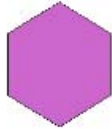
An irregular pentagon:



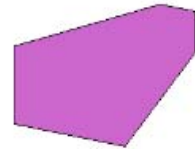
Hexagon

A six-sided polygon. The sum of the angles of a hexagon is 720 degrees.

A regular hexagon:



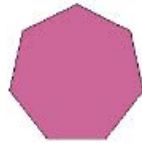
An irregular hexagon:



Heptagon

A seven-sided polygon. The sum of the angles of a heptagon is 900 degrees.

A regular heptagon:



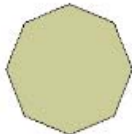
An irregular heptagon:



Octagon

An eight-sided polygon. The sum of the angles of an octagon is 1080 degrees.

A regular octagon:

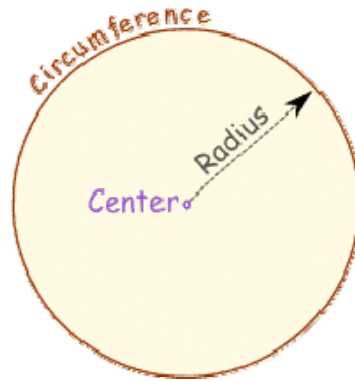


An irregular octagon:

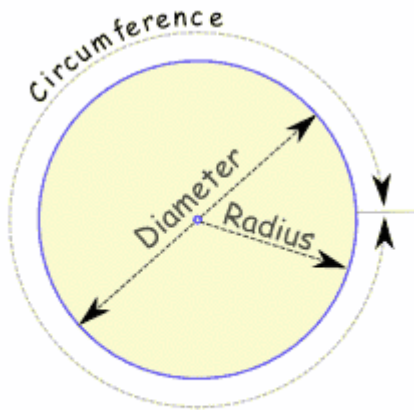
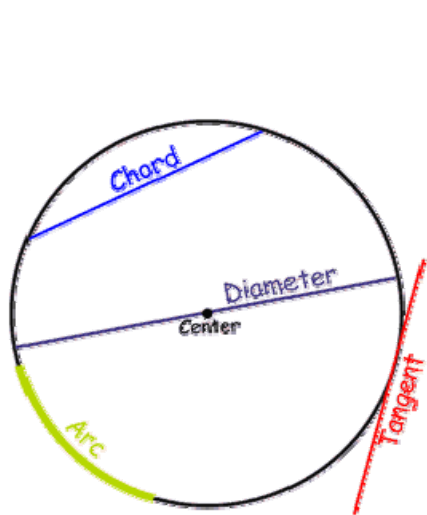


CIRCULAR SHAPES:

- Circumference: is a curved line in which every point is at the same distance from a fixed point called the centre.

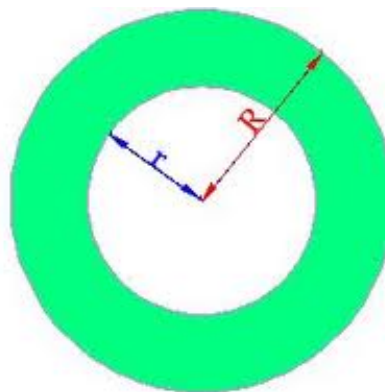
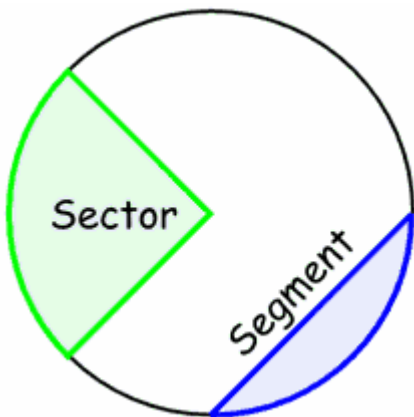


- Circle: is the zone inside a circumference.
- Lines of a circumference and number Pi:

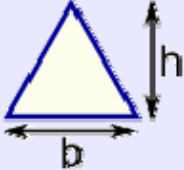
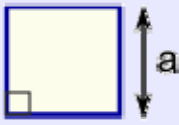

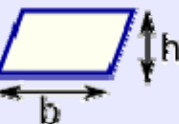
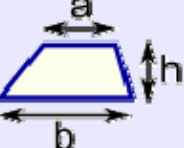

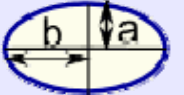



$$\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14159\dots$$

- Circular slices:



Area of Plane Shapes

	<p><u>Triangle</u> Area = $\frac{1}{2}b \times h$ b = base h = vertical height</p>		<p><u>Square</u> Area = a^2 a = length of side</p>
	<p><u>Rectangle</u> Area = $w \times h$ w = width h = height</p>		<p><u>Parallelogram</u> Area = $b \times h$ b = base h = vertical height</p>
	<p><u>Trapezoid (US)</u> <u>Trapezium (UK)</u> Area = $\frac{1}{2}(a+b) \times h$ h = vertical height</p>		<p><u>Circle</u> Area = πr^2 Circumference = $2\pi r$ r = radius</p>
	<p><u>Ellipse</u> Area = πab</p>		<p><u>Sector</u> Area = $\frac{1}{2}r^2\theta$ r = radius θ = angle in radians</p>

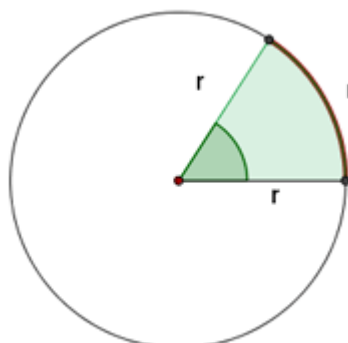
Units of Measuring Angles

A sexagesimal degree is the amplitude of the angle obtained by dividing the circle into 360 equal parts.

$$1^\circ = 60' = 3,600''.$$

$$1' = 60''.$$

A radian is the measure of an angle whose arc is one radius.



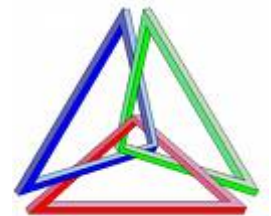
EXERCISES

1. Write the correct word for each definition:

- *Lines that never meet are.....lines*
- *A triangle with three equal sides is atriangle*
- *A quadrilateral with four equal sides and two pairs of equal angles is a*
- *The perpendicular bisectors of a triangle meet on a point called.....*
- *The heights of a triangle meet on a point called.....*
- *A angle measures more than 180° but less than 360°*
- *Lines that meet on one point are.....lines*
- *A triangle with two equal sides is antriangle*
- *A quadrilateral with four equal angles and two pairs of equal sides is a*
- *The medians of a triangle meet on a point called*
- *The bisectors of a triangle meet on a point called*
-

2. Classify these triangles taking into account their sides and their angles:

- *Two angles of 35° and 55°*
- *Two angles of 35° each one*
- *Two angles of 55° and 65°*
- *Two angles of 45° each one*
- *Two angles of 30° and 50°*



3. Find the measures of all the unknown angles in these polygons:

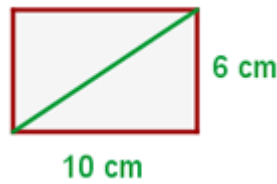
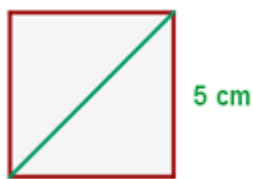
- *A rhombus with an angle of 130°*
- *An isosceles trapezium with an angle of 110°*
- *A rhombus with an angle of 130°*
- *An isosceles trapezium with an angle of 110°*
- *A right triangle with an angle of 40°*
- *An isosceles triangle with the different angle of 120°*

4. Classify these quadrilaterals:

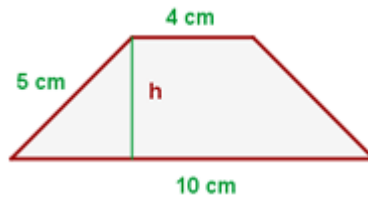
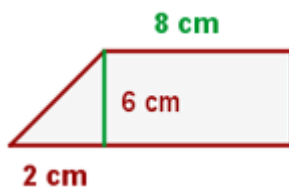
- *Four equal sides and four equal angles.*
- *Only opposite sides are parallel.*
- *Four equal angles and two pairs of equal sides.*
- *Four equal sides and two pairs of equal angles.*

5.-Solve these exercises about area and perimeter:

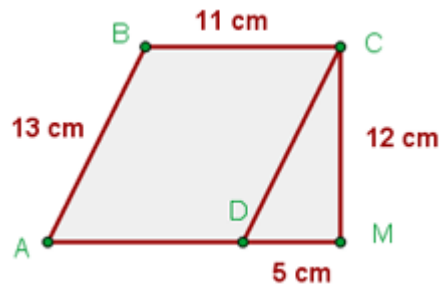
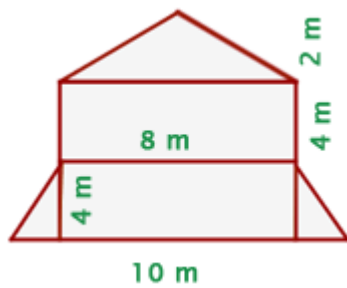
5.1 Get the perimeter and the area of this square and this rectangle:



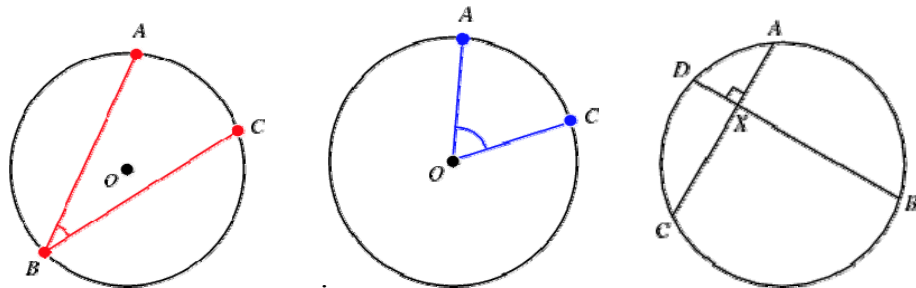
5.2 Get the perimeter and the area of these trapeziums:



5.3 Calculate the perimeter and the area of these shapes:



6. Think of one thing you can find with this shape:
- Circle.
 - Circumference.
 - Circular sector.
 - Circular crown.
7. Calculate the length of:
- A circumference of radius 5 cm.
 - An arc of circumference of radius 10 cm and of angle 30° .
8. In 100 turns of wheel with a radius of 70 cm, how far does it go?
9. Calculate the area of:
- A circle of radius 1 m.
 - A circular sector of radius 20 cm and a 45° angle.
 - A circular crown with radius 10 cm and 6 cm.
10. What is the radius of a circumference with a length of 43.96 cm?
11. What is the diameter of a circle with an area of 78.5 cm^2 ?
12. A CD has an exterior diameter of 12 cm, and an interior diameter of 1.5 cm. It is inside a sleeve (*funda*). Calculate the areas of both, the CD and its sleeve.
13. You have two donuts; One has radii of 4 cm and 7 cm, and the other one has radii of 8 cm and 10 cm. Which of them has a bigger area?
- 14.- Classify these angles:



Angles in the circumference:

inscribed equal to a half of the central angle of	half-inscribed equal to a half of the central angle	interior is measured by the semi-sum of arcs	exterior is measured by the semi-difference