

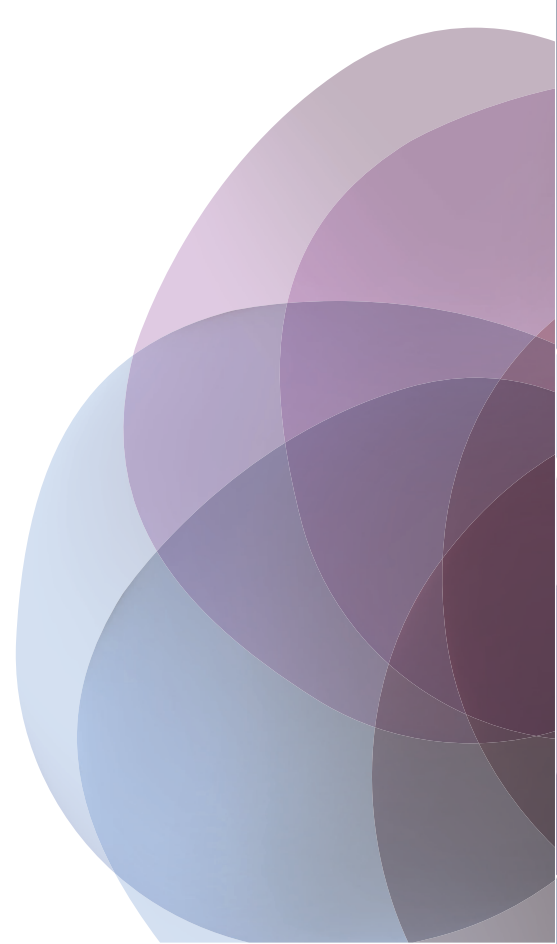
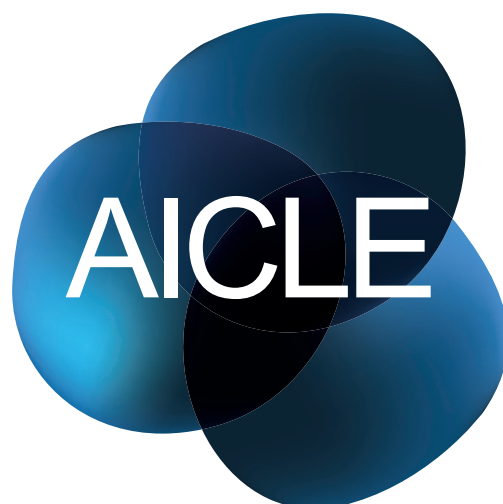
Matemáticas

Secundaria



JUNTA DE ANDALUCÍA

Inglés



Identificación del material AICLE

TÍTULO	Sequences
NIVEL LINGÜÍSTICO SEGÚN MCER	A2+
IDIOMA	Inglés
ÁREA / MATERIA	Matemáticas
NÚCLEO TEMÁTICO	Sucesiones
GUIÓN TEMÁTICO	<ul style="list-style-type: none">- Sucesiones de números reales: conceptos básicos.- Progresiones aritméticas y geométricas: definición, características y aspectos principales.
FORMATO	Material didáctico en formato PDF
CORRESPONDENCIA CURRICULAR	3º de Educación Secundaria
AUTORÍA	Cristina López Lupiáñez
TEMPORALIZACIÓN APROXIMADA	6 sesiones (más las necesarias para la post-task)
COMPETENCIAS BÁSICAS	<p>Competencia en comunicación lingüística</p> <ul style="list-style-type: none">- Conocer, adquirir, ampliar y aplicar el vocabulario del tema- Ejercitar una lectura comprensiva de textos relacionados con el núcleo temático <p>Competencia Matemática</p> <ul style="list-style-type: none">- Conocer las nociones básicas sobre probabilidad- Utilizar adecuadamente las estrategias del cálculo básico de probabilidades- Resolver situaciones utilizando las nociones matemáticas aprendidas <p>Aprender a aprender</p> <ul style="list-style-type: none">- Aprender a relacionar los conceptos tratados- Organizar las nociones, ideas y argumentos de forma ordenada y constructiva. <p>Autonomía e iniciativa personal</p> <ul style="list-style-type: none">- Ser autónomos para realizar las actividades individuales- Tener capacidad de juicio crítico ante opiniones ajenas- Expresar ideas propias de forma argumentada
OBSERVACIONES	Las actividades serán puestas en común y el profesor unificará conceptos y conclusiones, como corresponde a un aprendizaje constructivista. Un ejemplo de ello son las actividades sobre el crecimiento de las progresiones según la diferencia o la razón. Las actividades pueden complementarse con otras para mayor práctica de los procedimientos. Algunos de los contenidos se tratarán a partir de una puesta en común de cierta actividad.

Tabla de programación AICLE

OBJETIVOS	<ul style="list-style-type: none"> - Reconocer y plantear situaciones susceptibles de ser formuladas en términos matemáticos, elaborar y utilizar diferentes estrategias para abordarlas y analizar los resultados - Actuar ante los problemas que se plantean en la vida cotidiana de acuerdo con modos propios de la actividad matemática, tales como la exploración sistemática de alternativas, la precisión en el lenguaje, la flexibilidad para modificar el punto de vista o la perseverancia en la búsqueda de soluciones - Elaborar estrategias personales para el análisis de situaciones concretas y la identificación y resolución de problemas, utilizando distintos recursos e instrumentos y valorando la conveniencia de las estrategias utilizadas en función del análisis de los resultados y de su carácter exacto o aproximado 		
CONTENIDOS DE CURSO / CICLO	Análisis de sucesiones numéricas. Progresiones aritméticas y geométricas.		
TEMA	<ul style="list-style-type: none"> - Sucesiones de números reales - Término general de una sucesión - Sucesiones crecientes y decrecientes - Progresiones aritméticas y geométricas. Término general de las mismas - Suma de los primeros términos de una progresión geométrica o aritmética 		
MODELOS DISCURSIVOS	<ul style="list-style-type: none"> - Definir los conceptos básicos relacionados con las sucesiones y progresiones - Realizar afirmaciones sobre situaciones susceptibles de ser estudiadas mediante secuencias o sucesiones o mediante progresiones - Estimar la evolución de los términos de una sucesión - Discernir posibles nociones incorrectas preconcebidas sobre sucesiones y progresiones - Calcular términos generales y determinados de sucesiones-progresiones, y la suma de los primeros términos de una progresión aritmética o geométrica 		
TAREAS	<ul style="list-style-type: none"> - Problemas de cálculo - Presentación oral de problemas de secuencia - Proyecto final o post-task: The most famous sequeces 		
CONTENIDOS LINGÜÍSTICOS	FUNCIONES: Expresar probabilidad Describir el término en una secuencia	ESTRUCTURAS: I think it has to be / It's I got the same number because ... I think you are wrong because ... Don't you think there are more options? I do because ... This way we won't get anything useful because... We have to consider How do you calculate/ estimate/ define... this? Why do you think that?	LÉXICO: Sequence. (General) term. Length of a sequence. Increasing or (decreasing) sequence. Limit (of a sequence). Arithmetic/geometric progression. Common difference. Common ratio.
CRITERIOS DE EVALUACIÓN	<ul style="list-style-type: none"> - Conocer el significado de los principales conceptos tratados - Relacionar los conceptos para resolver situaciones prácticas y responder cuestiones sobre sucesiones - Calcular el término general de una sucesión, de una progresión aritmética, o de una progresión geométrica - Calcular términos determinados de sucesiones o progresiones - Calcular la suma de los primeros términos de una progresión aritmética o geométrica 		

A SEQUENCE OF IDEAS TO START...

Life is a list

In your daily life you can find a lot of lists. If you never thought about lists, it's high time to do so! Lists will never be the same after we finish these activities!

Work with other two-three students.



1) Order is the key.



Talk to your partner and discuss.

What do these situations have in common?



There are people waiting for their turn to buy some fruit.

You are looking for your name on a list because you want to know the mark you got on your last exam.

You have an assigned seat when you go to the movie theater.

Patient people don't get nervous if they have to stand in a queue for a long time.

You are trying to follow instructions, being careful to do every step of the process in order.

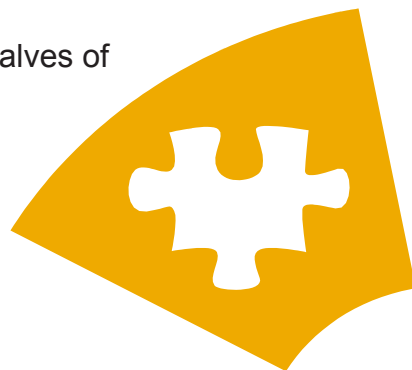
Now **compare your ideas with those** of your classmates!



2) Listen to your teacher and match the two halves of each sentence.



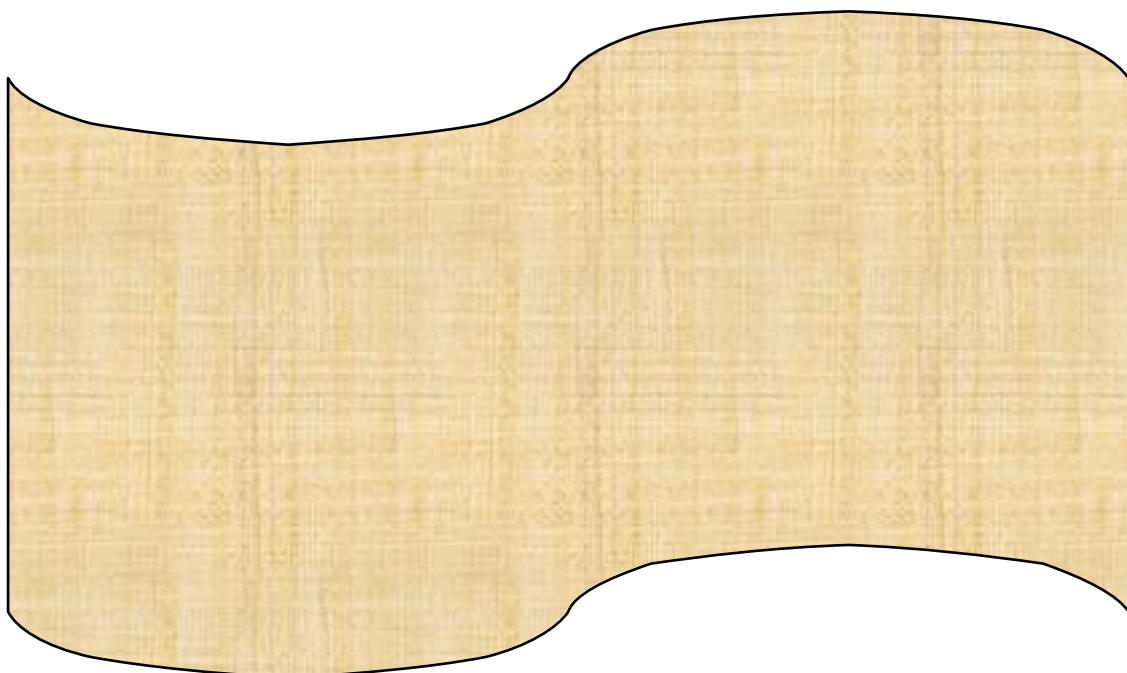
After that, write down any new vocabulary.



<i>The situations given before are related</i>	<i>list of things (objects, events...).</i>
<i>A sequence is an ordered</i>	<i>elements can appear several times at different positions in the sequence.</i>
<i>Like a set, it contains members (they can be called elements or terms), and</i>	<i>to a concept known as sequence.</i>
<i>Unlike a set, order matters, and the exact same</i>	<i>the number of terms (possibly infinite) is called the length of the sequence.</i>

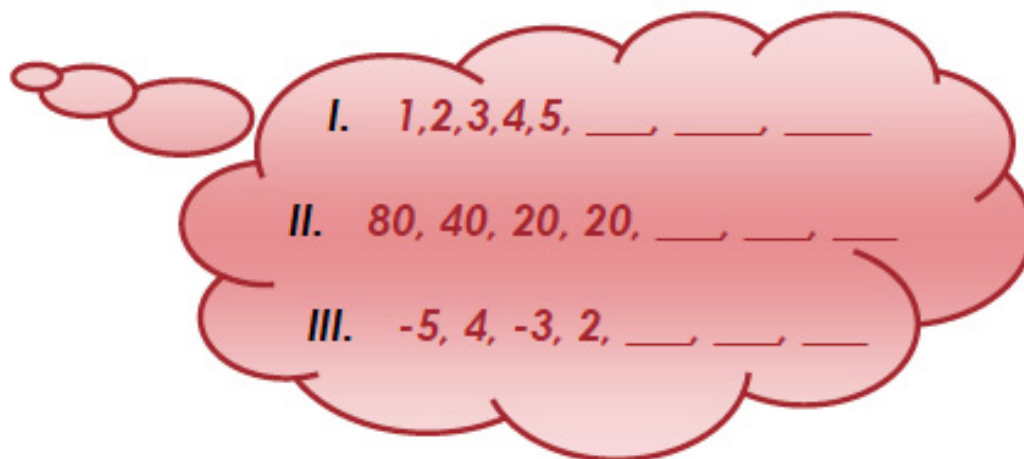


3) Describe three examples of sequences and compare your ideas with the rest of the group.



4) Guess the next term:

a) Write **three more terms** for the following lists. Do it yourself and then share your work with your group.

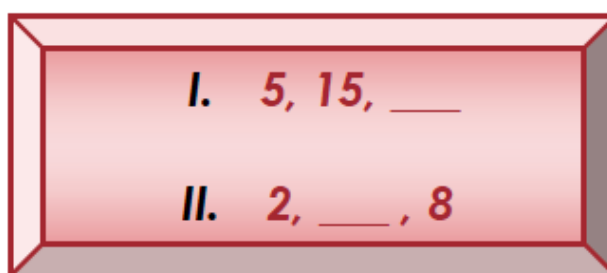


You can start like this:

- I think it has to be /It's ...
- I got the same number because ...
- I think you're wrong because ...
- Don't you think there are more options? I do because ...



b) Find the **missing number**, explaining your solutions to your group. Is there more than one possibility?



You can start like this:

- I think it has to be / it could be ...
- There are other solutions because ...





5) Write the five first terms of a sequence related to every situation:



a) Every month you go to the cinema twice as much as the month before but, you don't know when you won't have enough money to go anymore.

b) You want to study for two more hours every week, because last time you only studied for one hour every Tuesday and you failed all of your exams.

c) You have been growing three centimeters every year since you were 1.4m tall.



6) Good relationships!



a) Listen to your teacher Complete the sentences and write the correct sequence for every case.



Sentences (to complete)	Sequences (to write)
<ul style="list-style-type: none"> ▪ Every _____ is _____ the _____ and the sequence _____ by _____. ▪ The _____ term is _____ and every term is _____ ▪ The second term is the square root of sixteen and _____ ▪ Every term is the _____ of the _____ terms and the sequence start _____. 	



b) For every sequence write the relationship between each term and the previous one (how do you obtain a term from the previous one?).



◆ 5, 8, 11, 14, ...

◆ 4, 12, 36, 108 ...

◆ -20, 20, -20, 20, ...

◆ 9, 5, 1, -3, -7, ...

To obtain a term you have to... / Every term can be calculated by...



c) **The missing number game.**

We're going to play a game as a class! Here are the instructions:



- Each group writes a sentence describing how to obtain a term in a sequence.

- One group at a time reads the sentence to the rest of the class, and asks for a term to calculate.

- The first group to calculate the term gets two points and the other groups that are correct get one point.

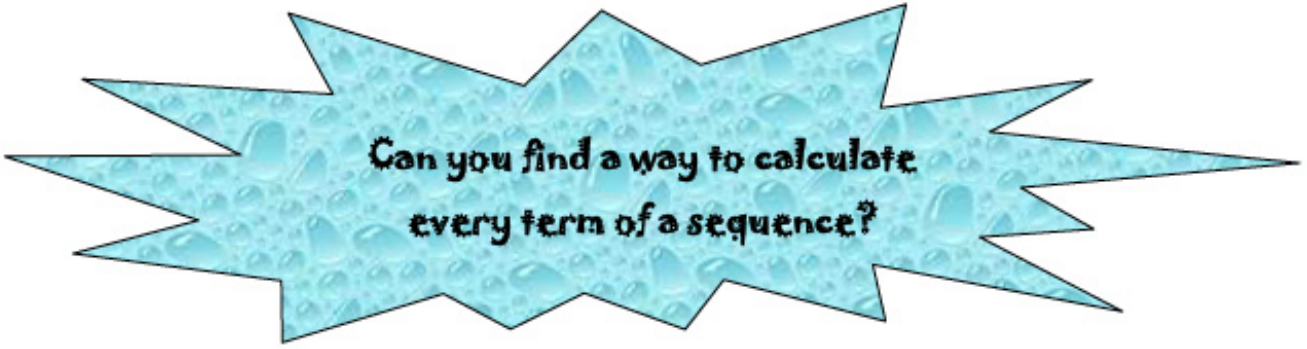


7) **To think about:**

In your groups think about the answer to the next question.



Then, your group will have to explain the answer to the rest of the class.



8) Use a formula to express the relationship between the value of every term and its position in the sequence. Work in groups!

$$\frac{+1}{-1} - \frac{1}{t+1} \Big) dt = 6(t^2 - t + \frac{1}{t})$$
$$t - Cn |E+1| + C =$$
$$\left[\frac{t^2}{2} + \sqrt{x} \cdot (n |t^2 x| + 1) \right] + C$$

a) 6, 12, 24, 48, ...

b) 3, 7, 11, 15, ...

c) 1, -1, 1, -1, ...

d) 4, 11, 16, 25, ...



The **general term** of a sequence is the expression that represents all the terms of the sequence (there are usually infinite terms!).

The general term is expressed by an algebraic expression that shows the relation between the value of a certain term in a sequence and the position of that term.



9) Calculate to win!

Your teacher will read the formulas of five sequences two times each and you will have to complete them. Then, a member of each group will get a specific term to calculate. Remember, the first group to give the right answer will get two points.



Here are the five sequences:

$a_n = \square n^2$	$b_n = \square + 4$	$c_n = -\square n$
$d_n = \frac{\square}{-3n}$	$e_n = \square n \square$	

Use this space to calculate, think, take notes and more:



"INCREASING" WHAT YOU KNOW

More about sequences

Sequences are very different. Some of them have certain properties that we can study. And some of them will take you to the limit! Work in pairs.



1) Guess the difference.

Look at these sequences:

- a) -50, -5, 360, 821, ...
b) 95, 70, 55, 41, -32, ...

Talk with your partner and choose the correct answer.



- ◆ **The third term of sequence b) is not correctly calculated because the formula is not being properly used.**
- ◆ **You cannot calculate the next term for sequence a) because the operation you have to do to calculate it is impossible.**
- ◆ **In a sequence a) every term is greater than the previous term.**
- ◆ **There is a mistake in one term of sequence a), it is not correctly calculated.**
- ◆ **On sequence b) every term is greater than the next one.**
- ◆ **Every sentence is correct.**

Speaking help: This one has to be true/false because...



2) **Listen** to your teacher and complete. After that, **write down** the new vocabulary in the box below.

A sequence is _____ if every term is _____ the _____
and a sequence is _____ if every term is _____ the _____.



New vocabulary:

3) Write the general term for and the three first terms of:

a) An **increasing** sequence.

b) A decreasing sequence.

c) A sequence that does **not increase or decrease**.

4) Analyzing a sequence (in pair):



a) Calculate at least **six consecutive terms** of the

sequence with the general term equal to $a_n = 3 + \frac{1}{n^3}$

b) Does the sequence **increase or decrease**?

c) What can you **learn by looking** at the terms you have calculated?

a) The six terms are:

b) The sequence...

c) We can see that the terms of the sequence...



We could say that the terms in the the sequence of last exercise “get close to 3”. If we consider higher and higher values for n (position of the term) we obtain terms that are closer and closer to 3. So we say 3 is the **limit** of the sequence.



5) Do it if you can!

Find the limit of these sequences... if you can!

$a_n = -3n$	$b_n = \frac{6n}{3n+1}$	$c_n = 2n - 0.5$
$d_n = n^2$	$e_n = (-1)^n$	$f_n = -5 \cdot \frac{5}{n^3}$

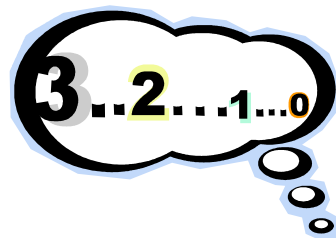


Work in pairs. Do your operations and write your conclusions:

Speaking help: We have to calculate... / The operation we have to do now is... / The limit has to be... / It's not possible to find a limit for this sequence because...

Special sequences

Some sequences are special. They have a particular structure, a special general term and singular characteristics. Work in group to find out what they are!



1) Investigate the difference...
Look at these sequences:



- a) 2, 5, 8, 11 ...
- b) 1, 2, 4, 7...

Work in groups and find out how the terms can be calculated in every case.

In sequence a) every term is calculated by...

In sequence b) every term is calculated by...

So the main difference is...

2) The special number is...


These sequences start with the same value: 5. But a “special number” makes the difference!


Match each sequence with the right “special number”:



Sequences

 5, 8, 11, 14, ...

 5, 11, 17, 23, ...

 5, 3, 1, -1, ...

"Numbers"

 -2

 6

 3

3) Progression!

It is time to learn what we call these special sequences and their magic numbers.

a) Listen to your teacher and work in groups to put the words in order. Then, write down the new vocabulary in the box below the text.

- Because they have are special Some sequences a singular structure.

- Terms of the sequence constant The difference of any two successive is a.

- Called common difference by the first term They can be defined and the constant value.

- Sequences These increase always (or decrease).

- Are arithmetic progressions These sequences called.



b) Select the arithmetic progressions and calculate the corresponding common difference:

2,4,6,8,10,

5,6,8,11,...

4,4.5,5,5.5,

3,7,11,13, ...

9,4,-1,-6, ...

The right arithmetic progressions are:	The common difference is:

c) Write three more terms for these sequences (d means common difference):



- $a_1=5, d=9$: _____
- $b_1=-3, d=2$: _____
- $c_1=0, d=0.3$: _____
- $d_1=8, d=-5$: _____



4) In groups. Decide if the following statement is true or false and say why. Share your ideas with the rest of the class.



The common difference of an arithmetic progression is related to its growth.

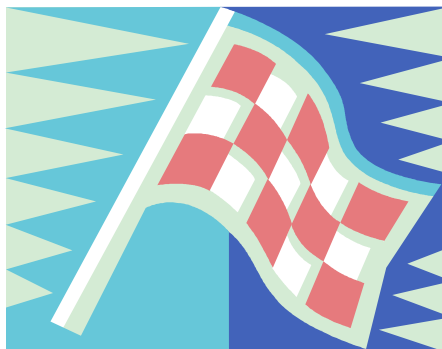
Speaking help:
- For group discussions: I think it has to be true if we think about.../ It's definitely false because in cases like...
- For sharing your ideas: We realized that the statement is true/false because as you can see...

5) The general term competition:

Each group will complete some questions and at the end the speaker will give the teacher a piece of paper with the final solutions.

Competition sheet questions:

Write the three first terms of the arithmetic progression with $a_1=2$ and common difference $d=3$:



How many “jumps” from term to term do you have to do to “arrive” from $a_1=2$ to a_{10} ?

How much do you “increase” or “decrease” with every “jump”?

How much do you “increase” in total from $a_1=2$ to a_{10} ?

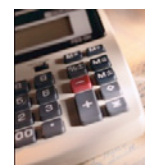
Calculate a_{10} :

Final question: the general formula to calculate the term a_n of an arithmetic progression with a_1 and d (common difference) given previously is...

The formula to calculate the term a_n of an arithmetic progression with the first term a_1 and d common difference d is...

6) Now practice:

Calculate the tenth term of these arithmetic progressions:



a) $a_1=70$, $d=150$ _____

b) $a_1=-67$, $d=95$ _____

c) $a_1=84.25$, $d=-53$ _____

d) $a_1=-2$, $d=-0.5$ _____

ANOTHER KIND OF PROGRESSION!



Special sequences... too!

There are more types of special sequences that you are going to discover here. Do the work in pairs.



1) Find out what the difference is...

Look at these sequences and discuss the questions.



- c) 2, 4, 6, 8, ...
- d) 2, 4, 8, 16, ...

- a) Which one is an **arithmetic progression**?
- b) Why is the second one **different**?

- a) The arithmetic progression is... because...
- b) The second one is different because instead...



2) Listen to your teacher and complete.

After that, each pair will write a sequence for every sentence, discussing and sharing their ideas and opinions.



Sentences (to complete)

Sequences (to write)

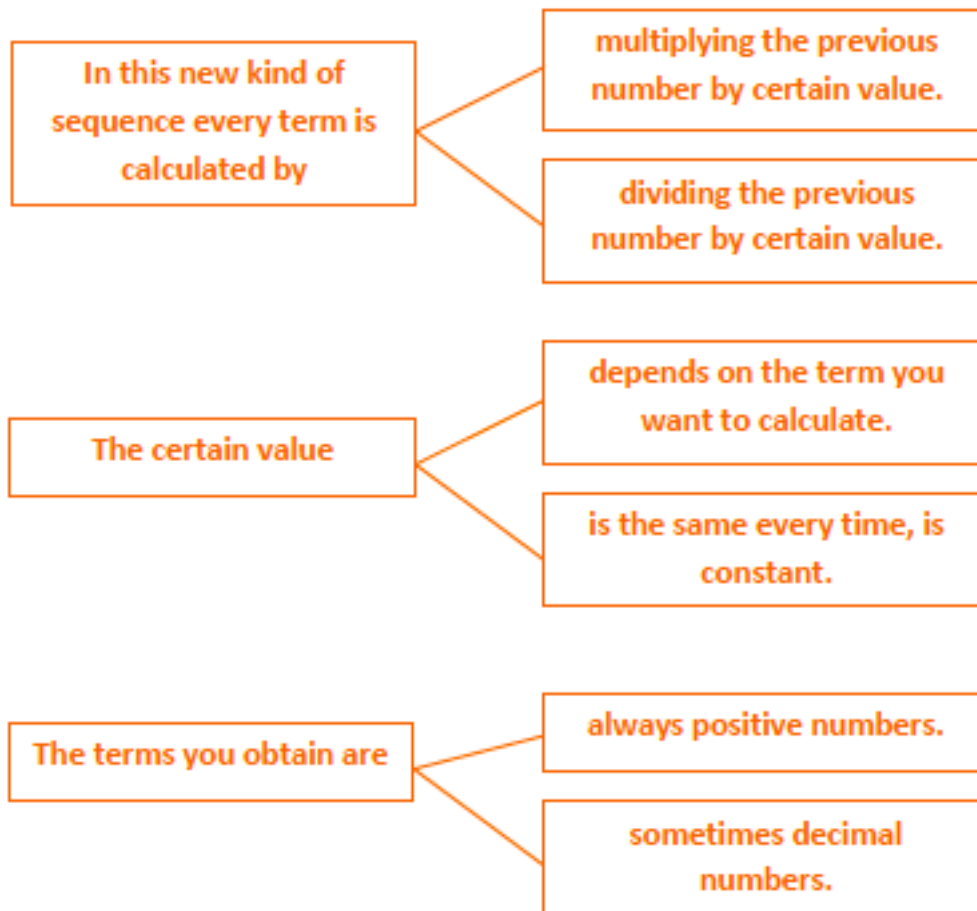
- Every ____ you recycle ____ times what you recycled last month. You started by ____ of garbage.

- Your _____ time is _____ every week since you studied _____ a week.
- Your business _____ every year _____ the profits you _____. At the beginning you earned _____.

3) Work with your partner to find the right option.



a) You have to **get to an agreement** so don't stop discussing until you get it!



Speaking help: It has to be / can't be this option because we saw an example where...



b) **Prepare an explanation** about your ideas about last question for you classmates.

Speaking help:

- We realized that according to the example... the right option is...
- We have considered that...
- If we analyze ...
- In summary we can say that...



4) **In pairs, complete the text with the words below.**
Then, write down the new vocabulary:



A _____ progression is a sequence where every _____ is calculated by _____ the previous one by a _____ number called the _____ of the progression.

Example: if $a_1 = 3$ and the _____ is $r=2$, the first terms are:

$$3, 3 \cdot 2 = 6, 6 \cdot 2 = 12, 12 \cdot 2 = 24 \dots$$

FIXED, COMMON RATIO(X2), GEOMETRIC, TERM, MULTIPLYING

5) **Match each geometric progression with the corresponding fifth term.**

First, you will need to find the common ratio!



Geometric progression	Fifth term
2, 10, 50, ...	-324
25, 75, ...	1250
-4, 12, -36, ...	0.25
4, 2, ...	2025



6) The general term competition II:

Each group will complete some questions and at the end the speaker will give the teacher a piece of paper with the final solutions.

Competition sheet questions:

Write the three first terms of the geometric progression with $a_1=2$ and common ratio $r=3$:

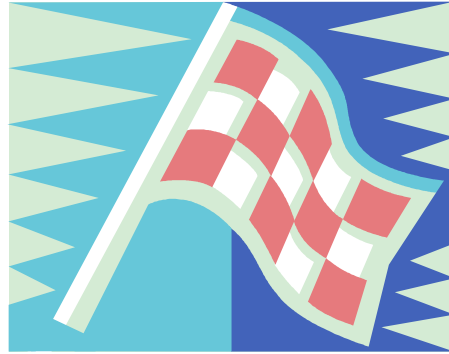
How many "jumps" from term to term do you have to do to "arrive" from $a_1=2$ to a_5 ?

What operation do you do every "step"?

How much do you "increase" in total from $a_1=2$ to a_5 ?

Calculate a_5 :

Final question: the general formula to calculate the term of a geometric progression with a_1 and r (common ratio) given previously is...



The formula to calculate the term of a geometric progression with first term a_1 and r common ratio r is... (Fill the gap with the last solution)

7) Now practice:

Calculate the tenth term of these geometric progressions:



- a) $a_1=7, r=5$ _____
 b) $a_1=6, r=9$ _____
 c) $a_1=-5, r=3$ _____
 d) $a_1=2, r=1/2$ _____



8) Think carefully about this:

The common ratio gives us information about the growth of the geometric progression.

With your partner, prepare a short presentation about the statement above.



Speaking help for your presentation:

- We considered some examples and we realized that...
- A geometric progression is increasing if...
- A geometric progression is decreasing if...
- A geometric progression is constant if...



We can do more!

We can study other things about arithmetic progressions (and geometric progressions later). Work in groups and you will see!



1) A special little boy.

In turns, read this text out loud. Make sure to speak loudly and clearly! After that, answer the questions below in groups.



Carl Friedrich Gauss was born on April 30, 1777 in Braunschweig, in what is now Lower Saxony, Germany. He was the son of poor working-class parents. There are several stories of his early genius. According to one, his gifts became very apparent at the age of three when he corrected, mentally and without fault in his calculations, an error that his father had made on paper while calculating finances.



Another famous story says that in primary school his teacher, J.G. Büttner, tried to occupy pupils by making them add a list of integers in arithmetic progression; as the story goes, these were the numbers 1 to 100. The young Gauss produced the correct answer within seconds, to the astonishment of his teacher and his assistant Martin Bartels.

We think that Gauss did this by adding the two terms from the opposite ends of the list which gave him identical intermediate sums: $1 + 100 = 101$, $2 + 99 = 101$, $3 + 98 = 101$, and so on, for a total sum of $50 \times 101 = 5050$. However, the details of the story are at best uncertain and some authors, such as Joseph Rotman in his book "A first course in Abstract Algebra", doubt that the story ever happened.

a) Who was Gauss? Where and when was he born?

b) What do you know about his family?

c) What did he do when he was only three years old?

d) Who were two of his first teachers?

e) Who wrote things about him? Is the legend true or not?

f) According to the legend what did he do one day in class?



2) What the boy did.

Do you want to repeat what little Gauss did? You are older than he was so you won't have any problem! Each group will complete the activities and give the final answers to your teacher. The winners will present their results to the class.



BUT this time, only one member of the group will do each activity, so it's A RELAY RACE!! Time's ticking....

Objective: calculate the sum of the 200 first whole numbers!!

a) Indicate the sum of the first 200 whole numbers.

b) Indicate the sum of the first 200 whole numbers in inverse order, below what your partner did.

c) Add the 200 “columns” your classmates did.

d) How much is double the sum you want to calculate?

e) FINAL ANSWER: Write a formula TO CALCULATE the sum of the 200 first whole numbers.



3) Let's investigate a bit more.

Gauss calculated the sum of the first 100 terms of a special arithmetic progression (the first term is $a_1=1$, the common difference is $d=1$) but...

What can you do to calculate the first 100 terms of a different arithmetic progression?



Calculate the 54 first terms of the arithmetic progression with $a_1=4$ and $d=5$

You should solve this problem first:
If $p+q = s+t$ does $ap+aq = as+at$?

Speaking help:

- We should start by...
- For the previous investigation we should... / we know that the affirmation...
- How can we use the previous investigation to...?
- This way we won't get anything useful because...
- So, finally we know that...

4) Listen to you teacher and complete the text.

The _____ to calculate the _____ of the first n terms of an _____ progression (S_n) is the result of doing this:

- _____ and the _____ you want to _____.
- _____ by the number of _____ you are _____.
- _____ everything by _____.

Example: if you want to add the 20 first terms of the arithmetic progression with first term $a_1=3$ and common difference $d=5$...

$$S_{20} = \frac{(a_1+a_{20}) \cdot 20}{2}; a_{20}=3+19 \cdot 5=98; S_{20} = \frac{(3+98) \cdot 20}{2} = \boxed{1010}$$

5) Time to practice: calculate and complete.



a_1	d	a_n	S_n
2	6	$a_{10} =$	
-15	86	$a_{157} =$	
	98	$a_{50} = 20$	

Calculations:

A HIGHER PROGRESSION...

More about geometric progressions!

If you thought we could do something similar to what we saw in the last section with geometric progressions...

You are right!



Work in pairs.



1) On the way

In some of the problems you will solve the formula needed to calculate the sum of the first “n” terms in a geometric progression.

Are you and your partner ready for the challenge?

a) Write the first five terms of the geometric progression with the first term $a_1=2$ and the common ratio $r=3$:

$$S_5 = 2 + 2 \cdot 3 + \text{---} \text{---} \text{---}$$

b) Multiply the previous expression by 3:

$$S_5 \cdot 3 = 2 \cdot 3 + 2 \cdot 3^2 + \text{---} \text{---} \text{---}$$

c) Carefully subtract both expressions:

$$S_5 \cdot 3 - S_5 = \text{---} \text{---} \text{---}$$

d) Write the formula:

$$S_5 =$$

So you have this:

The formula to calculate the sum of the first n terms of a geometric progression (S_n where the first term is a_1 and the common ratio is r) is:

$$S_n = \frac{a_n r - a_1}{r - 1}$$

Example: if you want to add the 12 first terms of the arithmetic progression with first term $a_1=2$ and common difference $r=3$...

$$S_{12} = \frac{a_{12}-2}{3-1}; a_{12} = 2 \cdot 3^{12-1} = 4096; S_{12} = \frac{4096-2}{2} = 2047$$

2) Practice:



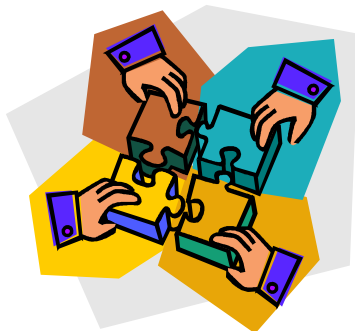
a_1	r	a_{10}	S_{10}
2	7		
-3	10		
5	1/3		

Calculations

WHAT ARISTOTLE SAID

Intelligence is more than knowledge.

Aristotle said that intelligence is the capacity of putting knowledge into practice. So let's put into practice what you learned while solving some problems.



Work in groups.

Note:

For every problem the process will be the same: each group will do it and share the solution with the rest of the class, in order to get a common answer.

1) Getting fit:

You have decided to start swimming every day, to get fit, because you know health is important. Today, you will only swim 150m. If you are determined to swim 50m more each day...

- How many meters will you swim by the end of next month?
- When will you be able to swim 2km?



2) Money:

An entrepreneur wants to study the evolution of the profits (€) of his enterprise during the past 12 months.

During the three first months the profits were 4 millions, 5.5 millions and 7 millions.

If the rate of increase of the profits remains the same:

a) How much money will he earn by the last month of the year?

b) Can you calculate the total profits for the whole year?



3) A vending machine:
Every day half of the drinks in a vending machine are sold. If the machine has a capacity of 320 drinks and is filled when there are 20 drinks left... How often does the owner of the machine need to fill it?



4) Nature:
There are two kinds of animals in a forest. At the beginning of the year 2005, there were 50 animals of both species (25 of each kind).

The first species doubles in population every year, and the population of the other one increases by 5 members during the same amount of time.

a) Calculate the population of both species in 2020.

b) Which population grows faster?



POST-TASK: THE MOST FAMOUS SEQUENCES.

In this project you learn about two of the most famous sequences ever discovered. It will be a very interesting investigation. Are you and your group ready?

You are going to do a project about:

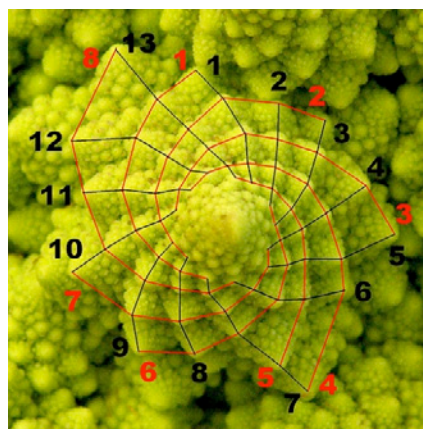
- ✓ The Fibonacci sequence
- ✓ The golden ratio
- ✓ The number e

Each group will write an essay and at the end, everyone will present his/her results. Each group will choose only one of the three sections to research. Your teacher will assign the sections according to the number of groups.

The sections for the essay are:

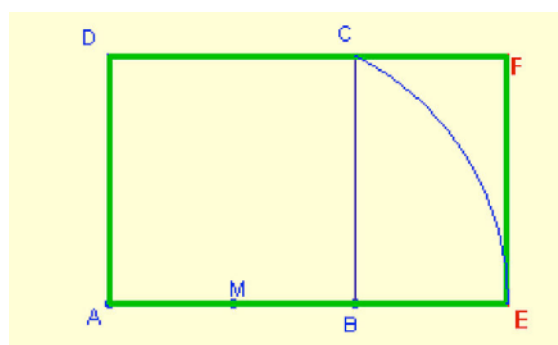
Part I: The Fibonacci sequence.

1. Leonardo of Pisa (Fibonacci):
biographical information.
2. The famous problem about rabbits,
rabbits and more rabbits.
 - The problem is...
 - The Fibonacci sequence.
3. Fibonacci numbers everywhere:
 - Fibonacci numbers in nature.
 - Fibonacci numbers in popular culture (cinema, books...).

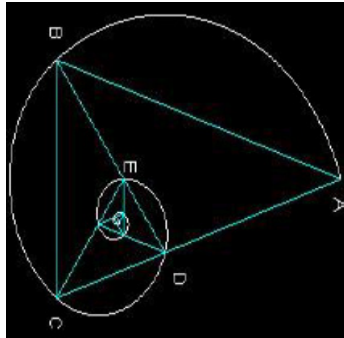


Part II: Golden ratio.

4. A special limit: if you divide every term of Fibonacci sequence by the previous one... what do you get?
5. The golden section and the golden rectangle.

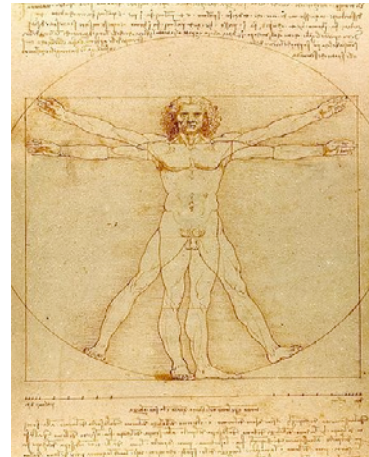


6. Durer spiral.



7. Golden ratio everywhere:

- Golden ratio in nature.
- Golden ratio in architecture.
- Golden ratio in music.
- Golden ratio in painting.



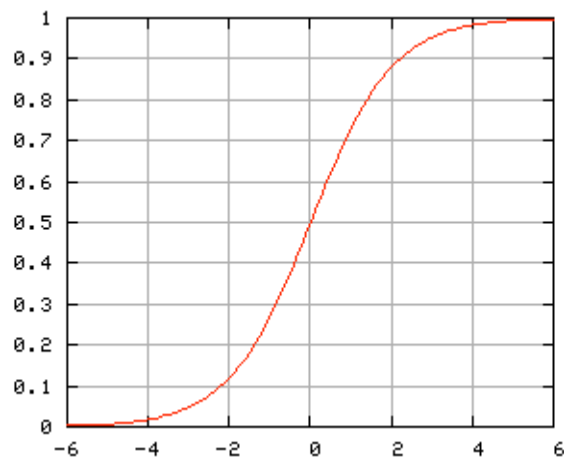
Part III: A number called e.

8. The sequence $a_n = \left(1 + \frac{1}{n}\right)^n$

- Write some terms from the sequence.
- Can you guess the limit of the sequence?

9. The e number: origin and history.

10. Applications and curiosities.



WHAT YOU LEARNED:

With this activities you have learnt...

That lots of things in life are related to sequences (and what a sequence is).

That some sequences have a general term that allows you to calculate every term of the sequence, according to its position.

That some sequences are increasing (or decreasing).

That there are special sequences, called arithmetic progressions, where the difference between two consecutive terms is a constant called a “common difference”.

That some sequences are called geometric progressions: Every term can be obtained multiplying the previous one by a constant called a “common ratio”.

That the general term for an arithmetic progression can be expressed by certain formula, and the same is true with a geometric progression.

That there is a formula to calculate the sum of the first “n” in terms of an arithmetic (or geometric) progression.

That sequences are important to solve certain real life problems. That there are very important sequences, that have been studied for a long time and that have several useful applications.

HOW WELL CAN YOU...?

Time to estimate how well you learned these topics



	ALWAYS	SOME-TIMES	NEVER
CONCEPTS			
I understand and remember the concepts that I studied about sequences.			
PRACTICE			
I can calculate terms of sequences using general terms, especially for arithmetic/geometric progressions. I can calculate the sum of the first n terms of a progression.			
LISTENING			
I understand when someone talks about sequences.			
READING			
I can read texts about situations related to sequences and understand the important information.			
SPEAKING			
I can talk about the main things related to sequences and their applications.			

WRITING			
I can describe situations where concepts related to sequences are involved.			
VOCABULARY			
I recognize words and expressions related to sequences.			

CREDITS

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http://commons.wikimedia.org/wiki/File:Karl_Friedrich_Gauss.jpg

http://commons.wikimedia.org/wiki/File:Math_2.png

http://en.wikipedia.org/wiki/File:Helianthus_whorl.jpg (by L. Shyama)

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