

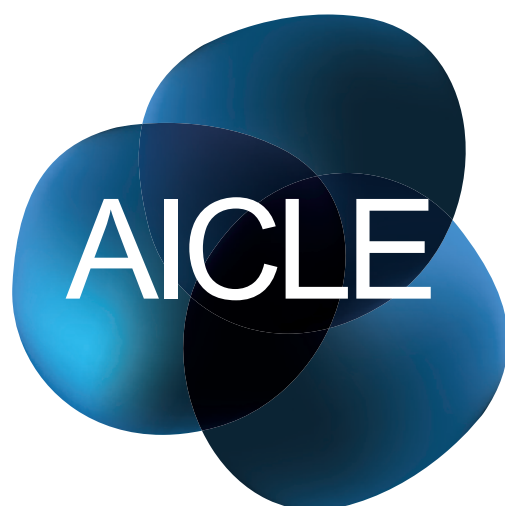
Matemáticas

Secundaria



JUNTA DE ANDALUCÍA

Inglés



Identificación del material AICLE

TÍTULO	The beginning: numbers
NIVEL LINGÜÍSTICO SEGÚN MCER	A2+
IDIOMA	Inglés
ÁREA / MATERIA	Matemáticas
NÚCLEO TEMÁTICO	Números
GUIÓN TEMÁTICO	<ul style="list-style-type: none"> - Tipos de números, operaciones - Aproximaciones. Radicales
FORMATO	Material didáctico en formato PDF
CORRESPONDENCIA CURRICULAR	3º de Educación Secundaria
AUTORÍA	Cristina López Lupiáñez
TEMPORALIZACIÓN APROXIMADA	Unas 10 sesiones (más las necesarias para las post-task)
COMPETENCIAS BÁSICAS	<p>Competencia en comunicación lingüística</p> <ul style="list-style-type: none"> - Conocer, adquirir, ampliar y aplicar el vocabulario del tema - Ejercitar una lectura comprensiva de textos relacionados con el núcleo temático <p>Competencia Matemática</p> <ul style="list-style-type: none"> - Conocer las nociones básicas sobre números - Distinguir los tipos de números y la relación entre ellos, realizar operaciones (radicales incluidos), aproximar y estimar errores de aproximación - Resolver situaciones utilizando las nociones matemáticas aprendidas <p>Aprender a aprender</p> <ul style="list-style-type: none"> - Aprender a relacionar los conceptos tratados - Organizar las nociones, ideas y argumentos de forma ordenada y constructiva <p>Autonomía e iniciativa personal</p> <ul style="list-style-type: none"> - Ser autónomos para realizar las actividades individuales - Tener capacidad de juicio crítico ante opiniones - Expresar ideas propias de forma argumentada
OBSERVACIONES	La unidad puede dividirse en dos. Es muy importante el tratamiento del vocabulario básico inicial (ver primera actividad) por ser la primera unidad del curso. Si se detecta carencia en el vocabulario básico (ver siguiente etiqueta) se debe incidir aún más en él. Puede ser útil completar las actividades propuestas con ejercicios para mayor práctica. Los alumnos/as necesitarán cintas métricas.

Tabla de programación AICLE

OBJETIVOS	<ul style="list-style-type: none"> - Reconocer y plantear situaciones susceptibles de ser formuladas en términos matemáticos, elaborar y utilizar diferentes estrategias para abordarlas y analizar los resultados - Actuar ante los problemas que se plantean en la vida cotidiana de acuerdo con modos propios de la actividad matemática, tales como la exploración sistemática de alternativas, la precisión en el lenguaje, la flexibilidad para modificar el punto de vista o la perseverancia en la búsqueda de soluciones - Elaborar estrategias personales para el análisis de situaciones concretas y la identificación y resolución de problemas, utilizando distintos recursos e instrumentos y valorando la conveniencia de las estrategias utilizadas en función del análisis de los resultados y de su carácter exacto o aproximado 		
CONTENIDOS DE CURSO / CICLO	<ul style="list-style-type: none"> - Números decimales y fracciones. Transformación de fracciones en decimales y viceversa. Números decimales exactos y periódicos. Fracción generatriz - Operaciones con fracciones. Cálculo aproximado y redondeo. Error absoluto y relativo. Utilización de aproximaciones y redondeos en la resolución de problemas de la vida cotidiana - Potencias de exponente entero. Significado y uso - Comparación de números racionales 		
TEMA	<ul style="list-style-type: none"> - Tipos de números: fracciones, decimales, números reales - Operaciones con fracciones. Potencias - Aproximación de números reales (truncamiento, redondeo). Errores - Radicales: significado y operaciones 		
MODELOS DISCURSIVOS	<ul style="list-style-type: none"> - Redactar argumentos y conclusiones - Argumentar respuestas - Exponer ejemplos, nociones e ideas - Expresar acuerdo o desacuerdo con las ideas de otros - Preguntar el por qué de ciertas afirmaciones 		
TAREAS	<ul style="list-style-type: none"> - Actividades para adquirir el vocabulario específico - Actividades para la comprensión significativa y relacional de los conceptos tratados - Ejercicios para practicar los procedimientos tratados - Proyectos: History of numbers. Number Π. 		
CONTENIDOS LINGÜÍSTICOS	FUNCIONES: <ul style="list-style-type: none"> - Reconocer la necesidad de los números y conocer la evolución de los mismos - Diferenciar los distintos tipos de números y distinguir la problemática que resuelve cada uno - Realizar cálculos (con fracciones, números irracionales, radicales...) - Estimar valores aproximados a uno dado. Estimar el error cometido - Discernir la conveniencia de la realización de una aproximación 	ESTRUCTURAS: <ul style="list-style-type: none"> I think it has to be / it can be ... I obtained the same number because ... I think you are wrong because ... Don't you think there are more options? I do because ... This way we won't get anything useful because... We have to consider How do you calculate/ estimate/ define... this? Why do you think that? 	LÉXICO: BASIC INITIAL VOCABULARY: the students should remember (see first activity) words related to: Numbers, ordinals, order, magnitudes of the figures (decimal numbers), mathematical operations (addition, subtraction, multiplication, division), equality of mathematical expressions... Vocabulary (this unit): Whole number, integer, rational-irrational-real number, opposite, proper/improper fraction, equivalent fraction, amplify/simplify a fraction, irreducible fraction, parenthesis, power, base, exponent, even-odd number, radical, similar radicals, root, index, radicand, decimal number, exact-recurring decimal number, period, prime, (to) approximate, absolute/relative error, (to) round, (to) truncate.
CRITERIOS DE EVALUACIÓN	<ul style="list-style-type: none"> - Conocer los diferentes tipos de números - Conocer y aplicar la relación entre fracciones y decimales - Realizar adecuadamente cálculos de diversa índole con números y fracciones (operaciones básicas, potencias, radicales...) - Realizar convenientemente aproximaciones numéricas (por truncamiento y redondeo) y estimar el error cometido al aproximar 		

NUMBERS... FROM THE BEGINNING

Very ancient numbers

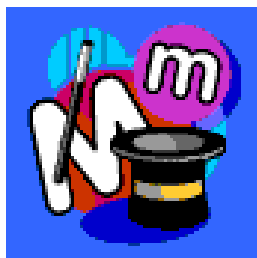
With the activities of this section you will realize two things: numbers are very (very!) ancient and numbers are everywhere. But before you will check how well you remember vocabulary you learnt in other courses.



Work in pairs!



1) What about your memory?



Let's play; you and your partner will be a team:

Prepare twelve pieces of paper. Write six expressions about numbers and operations with numbers in English. Fold the papers.

The game goes like this:

On your turn, you will choose three papers from the other teams.

For every paper you will give an example of the expression or calculate if it is an operation. You will get one point for every paper you do correctly.



2) Time ago...

When do you think numbers appeared? Read and you'll know. After reading the following text you and your partner will do a role play. One of you will be the teacher testing how well the other understood the text. After that, change your roles.

Text:

The origin of numbers is very, very ancient. The phenomenon of numbers appeared (probably) in different places around the world.

We could say they appeared because human beings needed to count things. But it is not a simple phenomenon, and until recently people of some tribes still counted using only “one”, “two” and “many”! At the beginning people counted using stones or notches on wood or bones.

One interesting example is the bone with 29 notches that was found in Lebombo (South Africa). It is a baboon bone and it is 37,000 years old. A wolf bone was discovered in Czechoslovakia, with 57 notches on it (divided into 5 groups of 11 and two more). This bone is 30,000 years old. These ancient bones are two of the most ancient counting objects ever found. Some people think they are related to the study of moon phases.

The origin of ordinals is supposed to be related to religious rites.

But... how did humans create signs to symbolize numbers?

The origin of written numbers is, like writing itself, associated with advanced societies. There were social structures where it was important to write down things about accountancy, taxes, properties...

Most ancient signs to represent numbers originated in ancient Mesopotamia: people used special signs that were written on little clay boards, used to count goods. Some of them are dated 8,000 B.C. Writing numbers was related not only to economy or trade, but also to agriculture (measuring fields). And numbers were used in astronomy too, to describe planets movements.

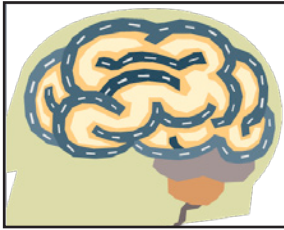


So you can see that the origin of civilization was possible because of the origin of numbers and... the origin of mathematics!!

Teacher: Student A	Teacher: Student B
Questions: When did numbers appear? Why did prehistoric people need numbers? What kind of numbers did prehistoric people use?	Questions: Is there any evidence about the origin of numbers? How did prehistoric people represent numbers? Are there other types of numbers that were used later?
Evaluation: my partner... Understood the text perfectly. Understood the main ideas in the text. Didn't understand the text very well.	Evaluation: my partner... Understood the text perfectly. Understood the main ideas in the text. Didn't understand the text very well.



3) Think carefully. Select the right options (in pairs).



For the wrong sentences give an example to show they are not right (write the examples in the box below).

- a) When you add two whole numbers you **ALWAYS** obtain a whole number.
- b) When you divide two whole numbers you **ALWAYS** obtain a whole number.
- c) When you subtract two whole numbers you **ALWAYS** obtain a whole number.
- d) When you multiply two whole numbers you **ALWAYS** obtain a whole number.

Examples:



Now do not forget to take notes in your vocabulary notebook: **WHOLE NUMBER(S)**



4) Old friends.

a) What is the main difference **between “5-3” and “3-5”**? Discuss with your partner and write a brief text about **the need for new numbers**.



The difference is...

We need other numbers because we cannot...



b) Now listen to your teacher and complete the text.

After that take note of the new vocabulary:



At the _____ people used _____ to _____ their goods and identify things, and they were able to _____ and _____ those numbers (obtaining _____ whole numbers). They could subtract when the minuend was _____ than the subtrahend. But people created other _____ of _____ to express debts and other _____ with other kinds of _____: if the _____ is _____ than the _____ the _____ is NOT a _____! These new numbers are called integers. The set of the integers _____ the whole numbers, the _____ and the opposite of every whole number (expressed by the _____ “-” before it, like -6). So the result of a sum/ multiplication/subtraction of two _____ is another integer, that can be zero (0), a _____ number (2, 6, 152...) or a _____ number (-24, -85, -652...).

Words to complete the text:

other, minuend, whole number/s, zero, situations, add, negative, subtraction, higher, less, kinds, numbers, beginning, subtrahend, result, multiply, contains, sign, count, integers, positive.

c) Now prepare a brief exposition for the rest of your classmates, about the following questions:

**Do we need other kind of numbers besides integers? Why?
What numbers do we need?**



Now ‘listen to your teacher and take note of the new vocabulary: RATIONAL NUMBER(S).

5) Rational numbers.



a) Match the two halves of the sentences.

First half of the sentence	Second half of the sentence
With a rational number you can express	by a fraction
You can represent a rational number	a numerator and a denominator
A fraction has two parts:	that represents an integer is 1
The denominator of the fraction	the division of two integers

b) Fractions in English.

You surely remember how to read and write fractions in English.....



(1) Listen to your teacher and complete the text, the words you (and your partner) have to use are given below.

A fraction has ____ parts: $\frac{3}{4}$

Numerator: it indicates the number of ____ we _____. This case the fraction means ____ fourths.

Denominator: it ____ the number of ____ parts in which we ____ the whole. This case we divide the whole into 4 parts. Every part is ONE ____, we use ordinal numbers.

If the denominator is equal to ____, the parts are not “seconds”, every part is a ____.
So $\frac{1}{2}$ is one half, $\frac{5}{2}$ means five ____, etc.

Another ____, especially if the numbers are ____, is to read the ____ using this formula: “numerator” ____ “numerator”, without using _____. So we can read $\frac{50}{30}$ this way: ____ over thirty.

Words to complete the text: equal, possibility, half, over, three, big, two, ordinals, divide, consider, fifty, parts, indicates, halves, two, fourth, fraction.

Word to take note: whole, numerator, denominator, fraction.

(2) Now practice:

- Everyone in class will read aloud the following fractions:

$$\frac{5}{2} ; \frac{91}{230} ; \frac{7}{3} ; \frac{27}{6} ; \frac{8}{10} ; \frac{65}{6301} ; \frac{12}{8}$$

- Listen to your teacher dictate some fractions. Write them in words and as fractions.

6) Equivalent fractions.

We are going to eat a pizza. Consider the pizza is the whole.

Be careful, don't eat too much!!



a) Talk to your partner **and select the pairs of fractions that represent the same amount** of pizza to eat. You can do pictures if you need.

$$\frac{1}{2} ; \frac{4}{8}$$

$$\frac{1}{5} ; \frac{2}{8}$$

$$\frac{-2}{3} ; \frac{-6}{9}$$

$$\frac{21}{15} ; \frac{14}{10}$$

$$\frac{11}{33} ; \frac{10}{32}$$

for you to take notes and do pictures:



b) Read carefully: take note of the new vocabulary.

Two fractions are **equivalent fractions** if they represent the same amount, the same number. We can check if two fractions are equivalent by doing the corresponding pictures to represent them.

But the best way to check it is to multiply "doing a cross", like this:

- $\frac{5}{4}$ and $\frac{10}{8}$ are **equivalent** fractions because **5x8** is equal to

$$\textbf{4x10: } \frac{5}{4} \quad \frac{10}{8}.$$

- $\frac{5}{3}$ and $\frac{10}{8}$ are **NOT equivalent** fractions because **5x3** is not

$$\text{equal to } \textbf{4x10: } \frac{5}{4} \quad \frac{10}{8}.$$

You can create fractions equivalent to certain one by multiplying or dividing denominator AND numerator by THE SAME number.

*When you multiply you **amplify the fraction** and when you divide you **simplify the fraction**.*

*If you simplify a fraction and obtain another one that cannot be simplified, this last fraction is called **irreducible fraction**.*

Most of the times to simplify a fraction is to find the irreducible equivalent fraction.

c) Use what you already learnt: the equivalent fractions game!



Every team will prepare **questions about equivalent fractions**.

On your turn, **ask one of the questions to the rest of the class...** the first team to answer correctly will **get one point**.

Are you ready to get points and be the winner??

Note: the questions will be similar to...

- Are these fractions equivalents or not? Why?
- Find a fraction equivalent to...
- Simplify the fraction...
- (...)



7) Everything in order! Who eats more pizza??



a) **Write a brief text** about how to order fractions. Some clues are given to help you. You will need to **talk to your partner** and share ideas.

Ordering fractions: how to distinguish the greater fraction?

- If two fractions have the **same denominator...**

- If two fractions have the **same numerator...**

- If the **numerators and denominator are not the same**, then...



b) Mary, Paul and John went to a restaurant to eat big sandwiches, pizzas and cakes.



When it was time to pay, they were confused.

Who should pay more?? This chart shows how much every one ate.

	Mary	Paul	John	Who had to pay more?
sandwich	$\frac{2}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	
pizza	$\frac{2}{8}$	$\frac{2}{5}$	$\frac{2}{6}$	
cake	$\frac{2}{8}$	$\frac{3}{9}$	$\frac{5}{12}$	



8) True or false?

Talk to your partner, share your ideas and decide if the following sentences are true or not.



You can do four mathematical operations with whole numbers obtaining another whole number.

You can do four mathematical operations with integers obtaining another integer.

You can do four mathematical operations with rational numbers obtaining another rational number.

Some integers are whole numbers.

Integers are not rational numbers.

Some rational numbers are integers.

Every integer can be expressed as a fraction without a denominator.

A fraction with a denominator equal to zero represents a whole number.

Speaking help: It is surely true-false because we know that.../ If it were true-false then.../ This example show us that...



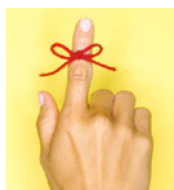
9) Order the words.



Put the words in order and rewrite the sentences, listening and working with your partner.



- is called A fraction when the denominator proper fraction the numerator is greater a than.
- when is less than the is called improper fraction numerator A fraction the an denominator.
- the unit (one) numerator and denominator are equal, the fraction When the represents.
- they represent are equivalent when the same number Two fractions (has the same result the division they represent).
- obtain equivalent fractions You can numerator and denominator by the same number by multiplying/dividing.



10) Remembering practical things!

a) Read the following examples to remember how to calculate with fractions. Discuss with your partner your questions (and ask your teacher if you need).

Remembering how to sum/subtract fractions:

a) Fractions with the same denominator:

- *To sum/subtract fractions with the same denominator all you have to do is to sum/subtract the numerators and maintain the common denominator.*

• Example: $\frac{3}{7} + \frac{9}{7} - \frac{15}{7} = \frac{3+9-15}{7} = \frac{-3}{7}$

b) Fractions with different denominators:

- *If the denominator is not the same, first you have to change the fractions for other representing the same rational numbers but with THE SAME denominator. After that you proceed as before.*
- *Example: $\frac{5}{6} - \frac{3}{4} + \frac{8}{3} = \frac{10}{12} - \frac{9}{12} + \frac{24}{12} = \frac{25}{12}$; 12 is the lowest common multiple of 6, 4, 3... that's our new common denominator and we operate with three equivalent fractions!*

Remembering how to multiply/divide fractions:

a) To multiply fractions all you have to do is multiply the numerators (to obtain the new numerator) and multiply the denominators (obtaining the new denominator).

- *Example: $\frac{-3}{5} \times \frac{6}{7} = \frac{(-3) \times 6}{5 \times 7} = \frac{-18}{35}$*

b) To divide fractions you multiply the numerator of the first fraction by the denominator of the second one (so you obtain the new numerator) and multiply the denominator of the first fraction by the numerator of the second one (obtaining the new denominator).

- *Example: $\frac{-3}{5} : \frac{6}{7} = \frac{(-3) \times 7}{6 \times 5} = \frac{-21}{30}$; and if you simplify the final result is $\frac{-21:3}{30:3} = \frac{-7}{10}$.*



Note: When there are mixed operations you do first parenthesis, multiplications/divisions and finally additions/subtractions. Simplify the result!

Vocabulary to take note: **PARENTHESIS, SIMPLIFY (FRACTIONS).**

b) Time to calculate: practice! First read these equations aloud and then solve them with your partner.

$$\left(3 + \frac{1}{4}\right) - \left(2 + \frac{1}{6}\right) =$$

$$\frac{1}{2} : \left(\frac{1}{4} + \frac{1}{3}\right) =$$

$$\left(\frac{5}{3} - 1\right) \cdot \left(\frac{7}{2} - 2\right) =$$

$$\left(\frac{3}{4} + \frac{1}{2}\right) : \left(\frac{5}{3} + \frac{1}{6}\right) =$$

$$\frac{2}{3} : \left[5 : \left(\frac{2}{4} + 1\right) - 3\left(\frac{1}{2} - \frac{1}{4}\right)\right] =$$



Power is yours!



a) What do you remember about **powers**?

Let's check and after the review you will calculate powers of fractions.



Complete the sentences. After that, a member of every pair will say aloud one answer. Wait for your turn and do it well!



A power has 2 parts, called _____ and _____

The base means _____ and the exponent means _____



To read a power you use ordinal numbers. Example: you read 34 as _____

If the exponent is 2 you say _____ and if the exponent is 3 you say _____

If the base is negative and the exponent is even, the result is _____. If the exponent is odd the result is _____.

If the exponent is negative _____



Take note: **power, base, exponent, even, odd.**

Calculate the value of the following powers. Be careful with the sign, don't get confused.

$$5^3 =$$

$$-4^5 =$$

$$(-20)^3 =$$

$$15^6 =$$

$$-51^4 =$$

$$(-85)^2 =$$

$$5^{-3} =$$

$$-25^{-7} =$$

$$(-40)^4 =$$



b) Properties of powers.

Complete every example and guess the corresponding general property.

Example	Property
$3^2 \cdot 5^2 = (\square \cdot \square)^2 = \square$ $3^2 : 5^2 = \square$	
$3^2 \cdot 3^5 = 3^{2+} \square$ $3^9 : 3^5 = 3^{9-} = \square$	
$(7^3)^5 = 7^{x} = 7^{x}$	
$5^0 = \square ; 5^1 = \square$	

c) Powers of fractions: write three fractions. Calculate the third and fourth power of the fractions.

d) Read the text and write a brief summary. Listen to your teacher to complete the last paragraph.

Legend of the Ambalappuzha Paal Paayasam

According to the legend, Lord Krishna once appeared in the form of a sage in the court of the king who ruled the region and challenged him to a game of chess (or chaturanga). The king, being a chess enthusiast himself, gladly accepted the invitation. The prize had to be decided before the game and the king asked the sage to choose his prize in case he won. The sage told the king that he had a very modest claim and being a man of few material needs, all he wished was a few grains of rice. The amount of rice itself would be determined using the chess-board in the following manner. One grain of rice would be placed in the first square, two grains in the second square, four in the third square, eight in the fourth square and so on. Every square will have double the number of grains of its predecessor.

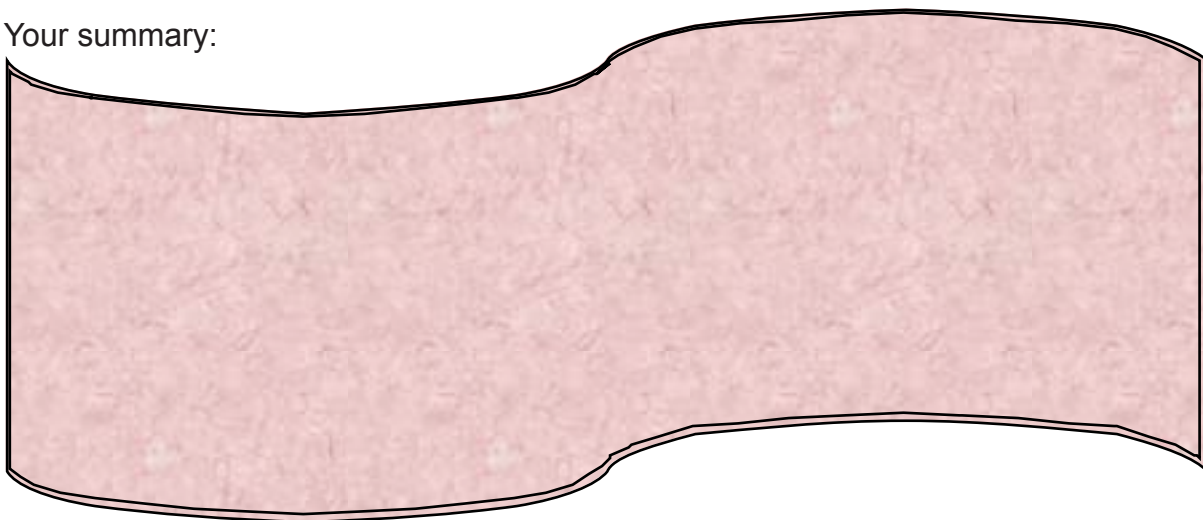
Upon hearing the demand, the king was unhappy since the sage requested only a few grains of rice instead of other riches from the kingdom which the king would've been happy to donate. He asked the sage to add other items to his prize but the sage declined.

So the game of chess started and needless to say the king lost the game. It was time to pay the sage his agreed-upon prize. As he started adding grains of rice to the chess board, the king soon realised the true nature of the sage's demands. By the 20th square, the number had reached one-million grains of rice and by the 40th square, it became one-trillion. The royal grainery soon ran out of grains of rice. The king realised that even if he provided all the rice in his kingdom and his adjacent kingdoms, he would never be able to fulfill the promised reward. The number of grains was increasing as a geometric progression and the total amount of rice required to fill a 64-squared chess board is... $2^{64}-1$, which is equal to 18446744073709551615 grains (about 18×10^{18} , or 18 billion billion grains). This amount of rice would weigh about 460×10^{12} kg, or 460 million billion tonnes (1,000 grains of rice weigh about 25g. This amount of rice would also cover the surface of India two meters deep!

Upon seeing ____dilemma, the sage appeared to the ____ in his true-form, that of ____ Krishna. He ____ the King that he didn't ____ to ____ the debt immediately but could pay him over _____. The king _____ paal-payasam (_____ of _____) in the _____ freely to the pilgrims every _____ until the _____ is _____ off.



Your summary:



12) Decimal numbers and fractions:



To express a rational number you can use a fraction... or a decimal number.

a) For every fraction write the corresponding decimal number.

FRACTIONS	DECIMAL NUMBER
$\frac{2}{4}; \frac{5}{8}; \frac{7}{5}; \frac{3}{125}$	
$\frac{9}{10}; \frac{3}{20}; \frac{18}{100}$	
$\frac{13}{6}; \frac{5}{24}; \frac{23}{12}$	
$\frac{52}{21}; \frac{20}{9}; \frac{4}{77}; \frac{10}{13}$	

b) Not the same type. Let's classify!

There are different types of decimal numbers.

A decimal number can be...

- ... an exact (or terminating) decimal.
- ... a recurring decimal, with a periodic part (period).
- Or another kind of decimal.

Look at the decimal numbers you obtained in last exercise and guess what kind they are!!

c) Investigate to be a magician!

You can impress your friends by telling them what kind of decimal number you will obtain from a fraction WITHOUT DOING THE DIVISION.

Write in pair the answers to the following questions and you will find how to be the greatest magician of decimals!!



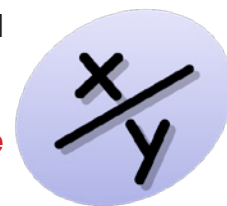
- **Write the denominators of the fractions (7a them in the same way as) (group them in the same way as the fraction are separated). Write the prime factors of the denominators.**

- **Write the kind of decimal corresponding to every group of fractions.**

- **How could you know what kind of decimal are you going to obtain from a fraction WITHOUT DOING THE DIVISION?**

d) Finding the lost fraction.

Solve the mystery; find the fraction that “originates” the decimal number. Here you have the information you need:



• **If a decimal number is exact, the corresponding fraction is like this:**

- o The numerator is formed by the digits (without the decimal point).
- o The denominator is the number formed by “1” and as many zeros as decimal figures the number has.

• **If a decimal number is recurring, the corresponding fraction is like this:**

- o The numerator is the difference between the number formed by the figures including the period (once) and the number formed by the figures BEFORE the period.
- o The denominator is the number formed by as many 9 as decimal figures the period has.
- o If the numerator is a decimal number, the fraction has to be multiplied by ten as many times as you need.

Now practice: write three exact decimals and three recurring decimals and calculate their corresponding fractions.

Space to work:



Let stop for a while to check your vocabulary. Your teacher will say aloud words or short expressions and you have to write them down. Your team will get one point for every right answer... if you are the first!

Very different numbers



There are other kind of numbers... yes! We have been talking about things that you knew... but It is time to show you some things that are, surely, new for you.

Work in groups!

1) Telling a story.

a) Read carefully the text and put the paragraphs in order, with your group.

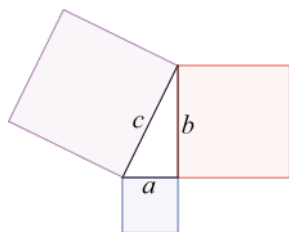


Pythagoras was an ancient Greek philosopher and mathematician. He was born on the island of Samos (570 BC.). He travelled widely when he was young and he went to Egypt and others places searching for more and more new things to learn.

They studied numbers deeply, doing interesting classifications, discovering important things about them. Their investigations are very important even today. You surely remember the famous

Pythagoras theorem (he was not the one who discovered it but he studied it deeply). The theorem is applied to right-angled triangles:

So C is a number that multiplied by itself equals 2. There is NO rational number like this mysterious C! The theorem was certainly proved as true, and then... what happened?



$$a^2 + b^2 = c^2$$

The sum of the areas of the two squares on

But there was a little problem. The numbers they adored were whole and rational numbers (divisions of whole numbers). They did NOT believe in other different kinds.

Around 530 BC he moved to Croton (Greece) and founded a religious sect. His followers were convinced that "numbers are everything". They believed that numbers represented magical things, it was a religion.

The problem was this: if you apply the Pythagoras Theorem for $a=1$ and $b=1$, you obtain

$$1^2 + 1^2 = c^2 \longrightarrow 1 + 1 = c^2 \longrightarrow 2 = c^2$$

It seemed "gods" (numbers) were playing with the faith of Pythagoras' followers. There was a great crisis. The monster had appeared: no rational numbers were there!! They called them "alogos" ("no sense" in Latin).



Now you are going to know them. Are you ready??



b) Now listen twice to your teacher and check if you were right.

c) Keep on **listening to your teacher**, you have double work: **first, complete the questions and then answer them.**

• was Who Pythagoras? he born Where was? _____

• did Where young he when he was go? _____

• Where move to when did he and? _____

• he there find did What? _____

• believe his followers What did he (and)? _____

• Which study did important theorem they? What did it establish? _____

• the was problem What? How did it affect them? _____

• What can you new numbers say about the? _____



Answers to the questions:

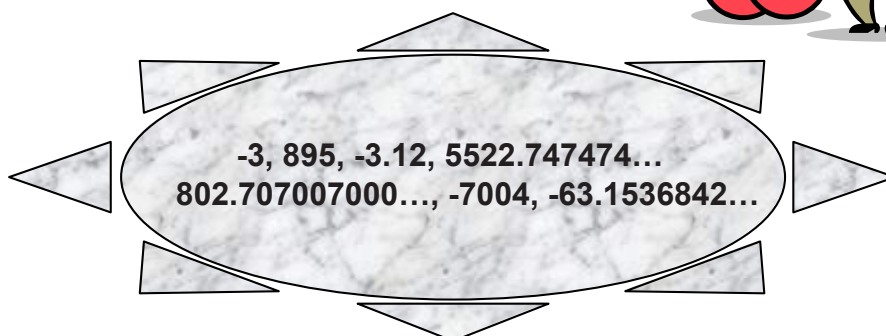
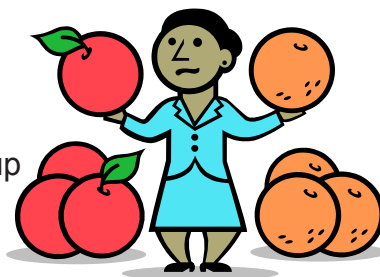




2) Rational or not?



a) Study the following numbers with your group. They are different **and you have to discover why**. Talk with your group and prepare an presentation for the rest of the class.



Speaking help: They are different because some of them... but others.../ We can appreciate that some numbers... but the other doesn't.



b) Read the text and say with your group if the expressions below are true or not. Take note of the new vocabulary.



There are different kinds of numbers; first you studied whole numbers and integers, with them you can add, multiply and subtract. After that you studied rational numbers, and division became possible.

But there are other operations that cannot always be done with rational numbers. Can you calculate the squared root of 3 obtaining a rational number? No, you can't so you need new "friends"!

You will recognize these new numbers because they have infinite decimal figures... which do NOT repeat!

This numbers were called "a logos" (or "with no sense" in Latin), now they are called **irrational** numbers.

True or false? Why?

- With integers you can divide numbers.

It's true/false because...

- With rational numbers you can do every mathematical operation.

It's true/false because...

- To divide numbers properly you need irrational numbers.

It's true/false because...

- Whole numbers and integers are irrational numbers.

It's true/false because...

- Some numbers can be rational and irrational.

It's true/false because...

- Irrational numbers have always infinite decimal figures.

It's true/false because...

- Rational numbers never have infinite decimal figures.

It's true/false because...

- Rational and irrational numbers have infinite decimal figures.

It's true/false because...



3) Finally...

Rational and irrational numbers are separated sets of numbers, and together they form a bigger set, the set of the real numbers.



Do a concept map about the different kind of numbers you know, and explain it to the rest of the class.

INTERESTING “MISTAKES”

A simple question to start...



Imagine you are measuring the height of a tree and a mountain. If you are 1m wrong measuring the mountain and 1 cm measuring the tree... when was your mistake worse?



You will realize this question is not as simple as it seems to be. Work in pairs, you will need your meter!

1) Your teacher will select a student of the class.



A member of every pair will measure the length of the student, as accurately as possible.

a) Every pair will share the measure with the class. **Did you all obtain the same value? If not, try to explain briefly why:**

b) Your teacher will establish the “true length”.

- Calculate the “mistake” you committed.
- What was your measure? Was it greater than the true value?



2) The error.

a) **The error game.** When you measure the length, weight, time, etc of something you almost never get the real value.

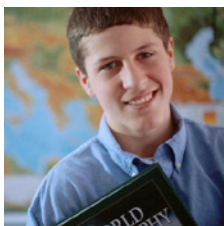


- Your team will measure the length of the sides of a **DIN-A4 sheet of paper** as accurately as possible. **Write down the values you obtained.**

- Your teacher will tell you the real measures of this kind of sheets. **Write down** the values.

- **Calculate** how much your own measures differ from the real values... so you calculate the “error” you made. Are your measures **less than the real** values? **If your error was less than the other YOU WIN!!**

b) **Read and learn:** read this explanation about numbers, measures and mistakes or “errors”. Talking about mistakes... there are some words which aren’t correctly written: find them.



We are glad to introduce you Peter:



The real length of Peter is 1.6057 m. But he always says his lngth is 1.60 m. This valu, 1.60 m, is an approximation for the rial value. As 1.60 m is not the real length there is an error of approximation.

The absolute error of the approximation can be calculted this way: we calculate the magnitude of the diference between the real value and the aproximation: $e = /1.6057-1.6057/= /0.0057/= \mathbf{0.0057\ m}$.

*In general: if **a** is an approximation of a real value **v** the absolute error of the approximation is $\mathbf{e= /a-r/}$. The error is expressed using the real value unti of measure.*

Note: do not forget to take note on your vocabulary notebook.

c) Your turn to practice: complete the chart working with your group.



Real value	Approximation	Absolute error
1h 5m 10s	1h 5m 7s	
	23.52 cm	0.04 cm
1456.76 km		0.26 km
30° 5' 2"	30° 5'	
50.20501 g	50.30 g	



3) Another error.



Peter has a little sister, Rose; sure you want to know her...
She is going to help you to understand more things about errors.



*The real length of Rose is 0.56 m. But Peter always says his sister is 0.60 m. So the absolute error this time is $e_{Rose} = 0.56 - 0.6 = -0.04$ m. **=0.04 m.***

The absolute error for Peter's length was $e_{Peter} = 0.0057$ m. It seems Peter is more accurate talking about his sister... ¿or not? Although e_{Peter} is greater than e_{Rose} , Peter did a bigger mistake with Rose's length... because things are not so simple.

If we consider the "amount" of error corresponding to every meter measured, we have the relative error of the approximation(r). It is very useful to compare measures and check how accurate we really were. For Peter and Rose we have these relative errors:

$$r_{Peter} = \frac{\text{absolute error}}{\text{real value}} = \frac{0.0057}{1.6057} = 0.00354 \dots$$

$$r_{Rose} = \frac{\text{absolute error}}{\text{real value}} = \frac{0.04}{0.56} = 0.0714 \dots$$

Peter is more accurate talking about his own length. Amazing isn't it??



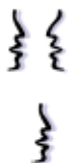
Answer these questions, discussing with your group:

- How many types of errors do you know? What do you call them?
- Can the absolute error be negative? And the relative error?
- Which unit do you use to express the absolute error? And the relative error?
- If you take different measures... how can you discover your more accurate approximation?

Answers after sharing ideas:



Calculate the relative errors for exercise 2)3).



4) Reasons to approximate.



Prepare with your group an answer to the following questions, after that you will share ideas with the rest of your classmates.

Why approximate? Can you calculate the errors if the real value is irrational?



5) The mountain and the tree. Now you can answer the question.



Imagine you are measuring the heights of a tree and a mountain. **Their respective real measures are 5051 m and 2.5 m.**

If you are 1m wrong measuring the mountain and 1 cm measuring the tree... when was your mistake worse?


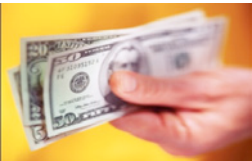


6) But how can you find the approximations?

Although an approximation (of a real value) is simply a value you choose instead of the real one, there are mathematical ways to find certain approximations.

a) Think about these situations, discuss with your group and guess the method used to approximate:

- We have approximated the number of people studying at a library.
- We have approximated how much a stone weighs.
- We have approximated how many cars are there in a parking.
- We have approximated how much money Paul earns every year.

	Real value	Approximation
People at a library 	157 people	160 people
Weight of the stone 	523.186 g	523.1 g
Number of cars 	3421 cars	3400 cars
Annual wage 	30752.65€	30753€

Speaking help: To approximate in this case we... but in the other example...

Write your ideas:



b) Listen to your teacher and complete the text. Work in group.

There are ____ important ways to approximate real numbers.

1) Truncation: **to approximate a number to the** tenths (units, tens, hundredths, thousandths...) by truncation is to omit the ____ at the right of the tenths. Last activity we truncated the weight of the stone to the ____.

2) Rounding: **to round a number to the** tenths (units, tens, hundredths, thousandths...) is to ____ an approximation by ____ this rule: you watch the figure at the ____ you want to approximate and...

- If the following figure is ____ than ____, you consider the following figures as zero. Last exercise we rounded **3421** people to the hundreds: as the following figure is 2 and it's less than five, the following figures are considered as ____: **3400** is the approximation.
- If the following figure is ____ or ____ than five, you add ____ (to the figure in the position you want to approximate to). Last exercise we ____ the annual wage **30752.65€** to the units: as the following figure is 6 and it's greater than five, we have to ____ 2 by 3 and the rest of the following ____ are considered as zero: **30753** is the approximation.



c) Practice to learn well!

Complete the chart to practice how to round numbers.

Real number	Round to the...	Approximation
215.574	units	
85.74	tenths	
9801.8656	thousands	



d) Explain why the following sentences are wrong and rewrite them properly. It may be useful to include examples. Do it in group.

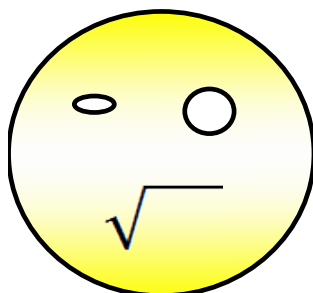
- When you truncate a number you always obtain an approximation by excess.

- You never obtain the same approximation by truncating and by rounding.

Radical for a while

Were you ok with powers? Sure you did, but it is time to get into a very radical world... the world of radicals.

Work in pairs!



1) Remember...



Remember the followers of Pythagoras had a problem. They needed a number C which satisfied $c^2=2$. As you remember c is **the squared root of 2**, $\sqrt{2}$, and it's not a rational number.



a) **Complete these sentences; you will need to calculate some simple things. After that, listen to your teacher to check you are right. Work in pairs** and remember to take note of the new vocabulary.



- The squared _____ of 9 is ____ because 3^2 is equal to _____.
- The _____ root of _____ is 10 because _____.
- The _____ of 144 is _____ because _____.

b) Calculate the square root of this numbers.

-25, 81, 196, -10000, 15876

Discuss with your group if you can do it in every case:

Sometimes we cannot calculate the squared root because...



2) Guessing how to say it.



Fill the gaps with the proper word.

Sure you can guess the magic missing one!

The _____ root of 125 is 5 because 5^3 is equal to 125

The _____ root of 16 is 2 because 2^4 is equal to 16

The _____ root of 243 is 3 because 3^5 is equal to 243

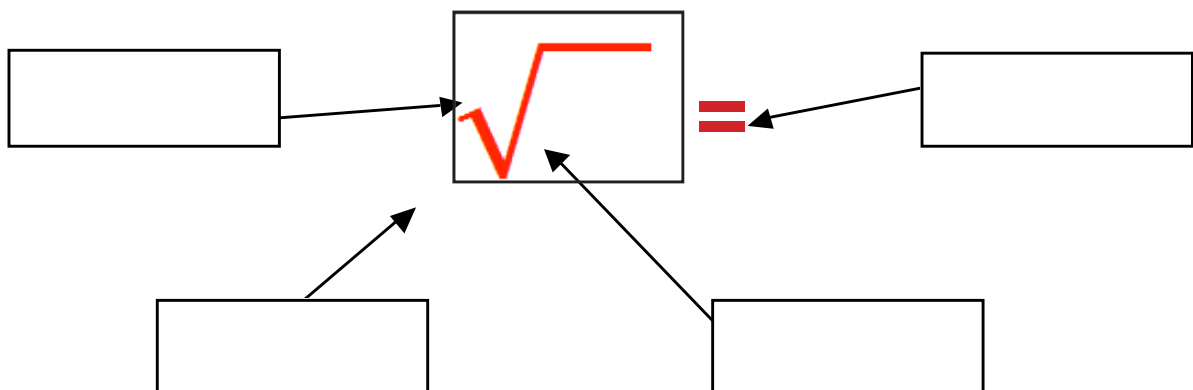
The _____ root of 1 is 1 because 1^6 is equal to 1

The _____ root of 128 is 2 because 2^7 is equal to 128



3) Some words:

a) **Choose the proper words:** fill the boxes with words from the list below.



LIST of words: **index, addend, factor, radicand, radical, sign, power, root**

b) Write the different parts of the expression: $\sqrt[3]{8}=2$.

Index	
Radicand	
Radical	
Root	



4) Going further...

a) Calculate the following roots:



$\sqrt{-16}$	
$\sqrt[3]{-27}$	
$\sqrt[5]{32}$	
$\sqrt[4]{625}$	

b) A bit more difficult:



$\sqrt{1296}$	
$\sqrt[3]{-3375}$	
$\sqrt[5]{-7776}$	
$\sqrt[4]{-62508}$	



c) Think about this question and prepare with your partner an exposition for the rest of the class:

How does the index indicate if you can calculate the root?



5) Aproximately...

Consider the radical: $\sqrt{12}$. The value of the root is not clear... it is not an integer. But you can guess an approximation.

a) Calculate an approximation for $\sqrt{12}$ (without decimal figures).

b) Use your **calculator to find the solution (rounding to the thousandths)**.

c) Repeat it for $\sqrt[3]{82}$; $\sqrt[3]{123}$; $\sqrt[3]{1012}$.

Work
board



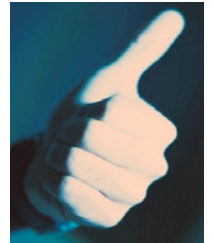
6) Powers and radicals.



Good with powers? Then it will be easy!

A radical can be expressed as a power whose exponent is a fraction.

Example: $\sqrt[5]{32^3}$ is equal to $32^{\frac{3}{5}}$



Write here some more examples:

Complete and listen to your teacher to check you and your partner were right:

Radical	Power
	$5^{\frac{3}{7}}$
$\sqrt[8]{9^{11}}$	
	$5^{\frac{1}{7}}$
$\sqrt[25]{52^{113}}$	
	$5^{\frac{3}{2}}$



7) Write an example for every property of radicals, work in group. After that your teacher will clarify how it works.

Example	Property
	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
	$(\sqrt[n]{a})^p = \sqrt[n]{a^p}$
	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$



8) **Operating with radicals:** let's play with radicals. If it seems difficult don't worry, work in groups and ask your teacher for help if you need it. All you need is practice!! Read the examples and do the activities.

- Simplifying radicals: $\sqrt[4]{25} = \sqrt[4]{5^2} = \sqrt{5}$
- Finding the common index. Which one is greater $\sqrt[3]{534}$ or $\sqrt{78}$?
 $\sqrt[3]{534} = \sqrt[6]{534^2} = \sqrt[6]{285\,156}$, $\sqrt{78} = \sqrt[6]{78^3} = \sqrt[6]{474\,552}$, so
- Extracting factors: $\sqrt{50} = \sqrt{25 \cdot 2} =$
- Gather radicals: $\sqrt{23} \cdot \sqrt{10} = \sqrt{230}$
- Thanks to all this stuff we can do things like...

$$\frac{\sqrt{5} \cdot \sqrt[3]{4}}{\sqrt[6]{40}} = \frac{\sqrt[6]{5^3} \cdot \sqrt[6]{4^2}}{\sqrt[6]{5 \cdot 2^3}} = \sqrt[6]{\frac{5^3 \cdot 2^4}{5 \cdot 2^3}} = \sqrt[6]{2 \cdot 5^2} = \sqrt[6]{100} = \sqrt[6]{10^2} = \sqrt[3]{10}$$

You can sum radicals ONLY if they are SIMILAR

- ▶ You cannot sum $\sqrt{5} + \sqrt{2}$ or $\sqrt[3]{5} + \sqrt{5}$
- ▶ But adding similar radicals is simple $3\sqrt{7} + 6\sqrt{7} - \sqrt{7} = 8\sqrt{7}$
- ▶ Sometimes you need tricks:

$$\begin{aligned} \sqrt{8} - \sqrt{50} + \sqrt{98} &= \sqrt{2^3} - \sqrt{2 \cdot 5^2} + \sqrt{2 \cdot 7^2} = \\ &= 2\sqrt{2} - 5\sqrt{2} + 7\sqrt{2} = 4\sqrt{2} \end{aligned}$$



New vocabulary: **similar radicals**

Activities to make radicals good friends of yours!



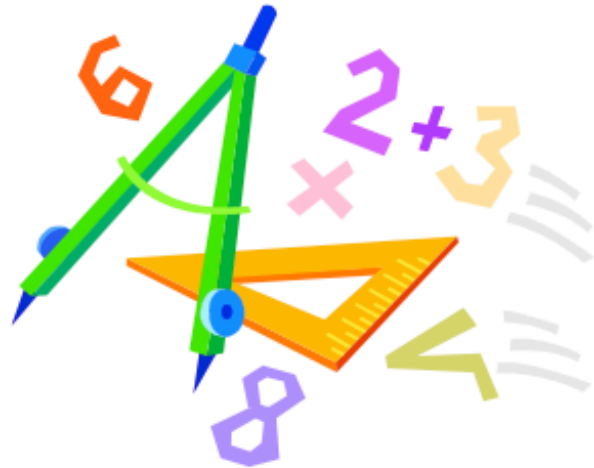
1) Calculate this powers:

1) $16^{\frac{3}{2}} =$

2) $8^{\frac{2}{3}} =$

3) $81^{0.75} =$

4) $8^{0.333...} =$



2) Extract factors:

1) $\sqrt{2 \cdot 3^2 \cdot 5^5} =$

2) $\sqrt[4]{2^7 \cdot 3^{14} \cdot 5^4} =$

3) Introduce factors:

1) $2\sqrt{3} =$

2) $2^2 \cdot 3^3 \sqrt[4]{6} =$



4) Find the common index:

$\sqrt{2}$

$\sqrt[3]{2^2 \cdot 3^2}$

$\sqrt[4]{2^2 \cdot 3^3}$

5) Sum the radicals:

1) $2\sqrt{2} - 4\sqrt{2} + \sqrt{2} =$

2) $3\sqrt[4]{5} - 2\sqrt[4]{5} - \sqrt[4]{5} =$

3) $\sqrt{12} - 3\sqrt{3} + 2\sqrt{75} =$

4) $\sqrt[3]{4} + \sqrt[3]{8} - \sqrt[12]{64} =$



6) Operate:

1) $2\sqrt{12} - 3\sqrt{75} + \sqrt{27} =$

2) $\sqrt{24} - 5\sqrt{6} + \sqrt{486} =$

3) $2\sqrt{5} + \sqrt{45} + \sqrt{180} - \sqrt{80} =$

4) $\sqrt[3]{54} - \sqrt[3]{16} + \sqrt[3]{250} =$



7) Do the addition:

1) $\sqrt{2} + \frac{1}{\sqrt{2}} =$

2) $\sqrt[3]{16} + \sqrt[3]{250} + \sqrt[6]{4} - \frac{1}{\sqrt[3]{4}} =$



8) Multiply the radicals:

1) $\sqrt{2} \cdot \sqrt{6} =$

2) $\sqrt{3} \cdot \sqrt[3]{9} \cdot \sqrt[4]{27} =$

3) $\sqrt{12} \cdot \sqrt[3]{36} =$

9) Do the divisions:

1) $\frac{\sqrt[5]{128}}{\sqrt[5]{16}} =$

2) $\frac{\sqrt[3]{4}}{\sqrt{2}} =$

3) $\frac{\sqrt{256}}{\sqrt[3]{16}} =$





VOCABULARY CHECKING!!



Your teacher will write on the board words you learnt... but

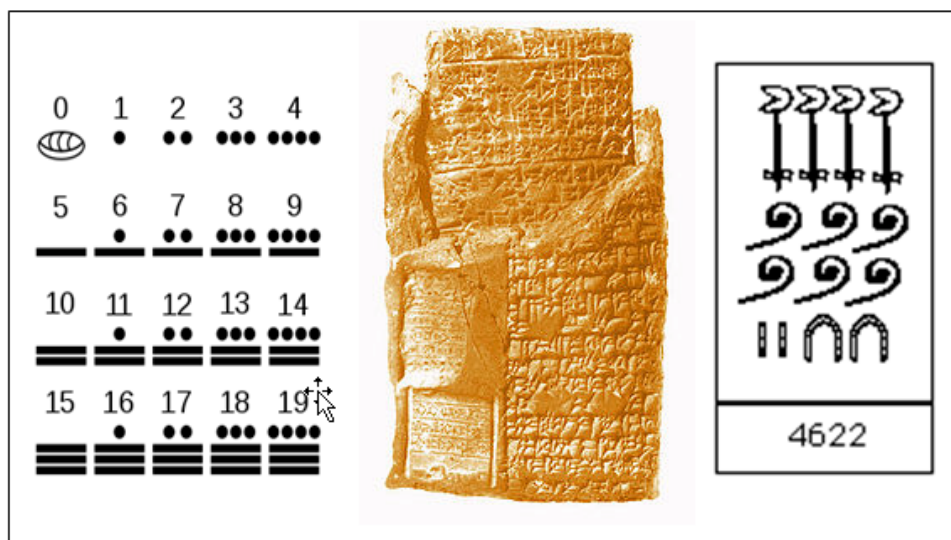
- Some letters will be disordered or...
- Some letters will be removed.

Your team will get one point for every guessed word.

POST-TASK-1: HISTORY OF NUMBERS.



With this project you will become an expert on history of numbers. You know something about the origin of numbers but it is time to learn more.



Prepare a presentation about THREE of these items:

- Ancient Mesopotamian numbers.
- Ancient Egyptian numbers.
- Ancient Roman numbers.
- Ancient Chinese numbers.
- Ancient Indian Numbers.
- Mayan culture numbers.



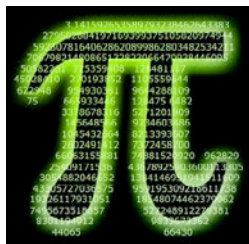
Focus on:

- The kind of numbers used.
- The operations people did.
- The problems people solved with numbers.
- Did fractions exist?



With this project you will know a lot about one of the most important irrational numbers in mathematics.

It is number π (Pi)



Investigate this special number:

- What is π ?
- How and when was it discovered?
- How much was π worth for ancient Egyptians?
- What's the value of π ?
- What kind of number is π ? Write three different approximations of Pi and estimate the absolute and relative errors.
- Include more things about this number on your own.

You can visit these websites to find some information:

http://www.dav.sceu.frba.utn.edu.ar/homovidens/chavezalejandro_mattje/trabajofinal/paginas/lahistoriadelnumeropi.htm
<http://webs.adam.es/rlllorens/pidoc.htm>
<http://www.sociedadelainformacion.com/fisica/pi/pi.htm>
http://www.portalplanetasedna.com.ar/numero_pi.htm

WHAT YOU HAVE LEARNT:

With these activities you have learnt...

- That development of numbers is related to human history.
- That there are different kinds of numbers, according to the need of new ones over time.
- The main concepts and operations about fractions.
- That fractions and decimal numbers are related.
- That sometimes we need to approximate measures or values, and we commit certain error.
- The meaning of radicals and how to operate with them.

HOW WELL CAN YOU...?



Time to estimate how well you learnt the topics.

	ALWAYS	SOME-TIMES	NEVER
CONCEPTS			
I understand and remember the concepts about numbers that I studied.			
PRACTICE			
I can calculate with fractions, powers, radicals...			
I can calculate approximations by rounding/truncation and the corresponding absolute/relative error.			
LISTENING			
I understand when someone talks about numbers and other related concepts.			
READING			
I can read texts about situations related to numbers and operations and understand the most important information.			
SPEAKING			
I can speak about the main things related to numbers/operations and their applications.			
WRITING			
I can write describing situations where concepts related to numbers are involved.			
VOCABULARY			
I recognize words and expressions related to numbers, operations, etc.			

CREDITS

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