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**On sums of dependent random lifetimes under the time-transformed exponential model**

P. 879-900

Jorge Navarro, Franco Pellerey, Julio Mulero

**Abstract**

For a given pair of random lifetimes whose dependence is described by a time-transformed exponential model, we provide analytical expressions for the distribution of their sum. These expressions are obtained by using a representation of the joint distribution in terms of bivariate distortions, which is an alternative approach to the classical copula representation. Since this approach allows one to obtain conditional distributions and their inverses in simple form, then it is also shown how it can be used to predict the value of the sum from the value of one of the variables (or vice versa) by using quantile regression techniques.

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**Preservation of distributional properties of component lifetimes by system lifetimes**

P. 901-930

Barry C. Arnold, Tomasz Rychlik, Magdalena Szymkowiak

**Abstract**

We analyze reliability systems with components whose lifetimes are identically distributed, and whose joint distribution admits a Samaniego signature representation of the system lifetime distribution. Our main result is the following. We assume that two systems have the same structure and that the lifetimes of the components of the systems share the same dependence copula. If the first system lifetime precedes (succeeds) its single component lifetime in the convex transform order, and if also the component lifetime of the second system precedes the (succeeds) component lifetime of the first system in the convex transform order then the system-component ordering property is preserved by the second system lifetime, i.e., the system lifetime precedes (succeeds) the component lifetime in the second system also. This allows us to conclude various sufficient and necessary conditions on the system signatures under which the monotone failure rate and density properties of the component lifetimes are inherited by the system lifetime under the condition that the component lifetimes are independent.

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**Testing conditional multivariate rank correlations: the effect of institutional quality on factors influencing competitiveness**

P. 931-949

Jone Ascorbebeitia, Eva Ferreira, Susan Orbe

**Abstract**

Joint distribution between two or more variables could be influenced by the outcome of a conditioning variable. In this paper, we propose a flexible Wald-type statistic to test for such influence. The test is based on a conditioned multivariate Kendall's tau nonparametric estimator. The asymptotic properties of the test statistic are established under different null hypotheses to be tested for, such as conditional independence or testing for constant conditional dependence. Two simulation studies are presented: The first shows that the estimator proposed and the bandwidth selection procedure perform well. The second presents different bivariate and multivariate models to check the size

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and power of the test and runs comparisons with previous proposals when appropriate. The results support the contention that the test is accurate even in complex situations and that its computational cost is low. As an empirical application, we study the dependence between some pillars of European Regional Competitiveness when conditioned on the quality of regional institutions. We find interesting results, such as weaker links between innovation and higher education in regions with lower institutional quality.

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**Precision matrix estimation using penalized Generalized Sylvester matrix equation**

P. 950-967

Vahe Avagyan

**Abstract**

Estimating a precision matrix is an important problem in several research fields when dealing with large-scale data. Under high-dimensional settings, one of the most popular approaches is optimizing a Lasso or  $\ell_1$  norm penalized objective loss function. This penalization endorses sparsity in the estimated matrix and improves the accuracy under a proper calibration of the penalty parameter. In this paper, we demonstrate that the problem of minimizing Lasso penalized D-trace loss can be seen as solving a penalized Sylvester matrix equation. Motivated by this method, we propose estimating the precision matrix using penalized generalized Sylvester matrix equations. In our method, we develop a particular estimating equation and a new convex loss function constructed through this equation, which we call the generalized D-trace loss. We assess the performance of the proposed method using detailed numerical analysis, including simulated and real data. Extensive results show the advantage of the proposed method compared to other estimation approaches in the literature.

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**Adaptive bi-level variable selection for multivariate failure time model with a diverging number of covariates**

P. 968-993

Kaida Cai, Hua Shen, Xuewen Lu

**Abstract**

In this study we propose an adaptive bi-level variable selection method to analyze multivariate failure time data. In the regression setting, we treat the coefficients corresponding to the same predictor variable as a natural group and then consider variable selection at the group level and individual level simultaneously. By imitating the group variable selection procedure with adaptive bi-level penalty, the proposed variable selection method can select a predictor variable at two different levels allowing different covariate effects for different event types: the group level where the predictor is important to all failure types, and the individual level where the predictor is only important to some failure types. An algorithm based on cycle coordinate descent is developed to carry out the proposed method. Based on the simulation results, our method outperforms the classical penalty methods, especially in removing unimportant variables for different failure types. We obtain the asymptotic oracle properties of the proposed variable selection method in the case of a diverging number of covariates. We construct a generalized cross-validation method for the tuning parameter selection and assess model performance using model errors. We also illustrate the proposed method using a real-life data set.

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**On the general  $\mathcal{E}$ -shock model**

P. 994-1029

Dheeraj Goyal, Nil Kamal Hazra, Maxim Finkelstein

**Abstract**

The  $\mathcal{E}$ -shock model is one of the basic shock models which has a wide range of applications in reliability, finance and related fields. In existing literature, it is assumed that the recovery time of a system from the damage induced by a shock is constant as well as the shocks magnitude. However, as technical systems gradually deteriorate with time, it takes more time to recover from this damage, whereas the larger magnitude of a shock also results in the same effect. Therefore, in this paper, we introduce a general  $\mathcal{E}$ -shock model when the recovery time depends on both the arrival times and the magnitudes of shocks. Moreover, we also consider a more general and flexible shock process, namely,

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the Poisson generalized gamma process. It includes the homogeneous Poisson process, the non-homogeneous Poisson process, the Pólya process and the generalized Pólya process as the particular cases. For the defined survival model, we derive the relationships for the survival function and the mean lifetime and study some relevant stochastic properties. As an application, an example of the corresponding optimal replacement policy is discussed.

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**Interquantile shrinkage in spatial additive autoregressive models**

P. 1030-1057

Jiawei Hou, Yunquan Song

**Abstract**

In this paper, we study the commonness of nonparametric component functions at different quantile levels in spatial additive autoregressive models. We propose two fused adaptive group LASSO penalties to shrink the difference of functions between neighbouring quantile levels. Using these methods, we can estimate the nonparametric functions and identify the quantile regions where functions are unvarying simultaneously. Therefore, when there exists a quantity-level region where the functions are unvarying, its performance is expected to be better than the conventional spatial quantile additive autoregressive model. Under some regularity conditions, the proposed penalized estimators can reach the optimal rate of convergence in theory and also recognize the true varying/unvarying regions accurately. At the same time, our proposed method shows good numerical results in simulated and real datasets.

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**Reducing degradation and age of items in imperfect repair modeling**

P. 1058-1081

Maxim Finkelstein, Ji Hwan Cha

**Abstract**

We develop new models for imperfect repair and the corresponding generalized renewal processes for stochastic description of repairable items that fail when their degradation reaches the specified deterministic or random threshold. The discussion is based on the recently suggested notion of a random virtual age and is applied to monotone processes of degradation with independent increments. Imperfect repair reduces degradation of an item on failure to some intermediate level. However, for the nonhomogeneous processes, the corresponding age reduction, which sets back the 'clock' of the process, is also performed. Some relevant stochastic comparisons are obtained. It is shown that the cycles of the corresponding generalized imperfect renewal process are stochastically decreasing/increasing depending on the monotonicity properties of the failure rate that describes the random failure threshold of an item.

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**Copula-based bivariate finite mixture regression models with an application for insurance claim count data**

P. 1082-1099

Lluís Bermúdez, Dimitris Karlis

**Abstract**

Modeling bivariate (or multivariate) count data has received increased interest in recent years. The aim is to model the number of different but correlated counts taking into account covariate information. Bivariate Poisson regression models based on the shock model approach are widely used because of their simple form and interpretation. However, these models do not allow for overdispersion or negative correlation, and thus, other models have been proposed in the literature to avoid these limitations. The present paper proposes copula-based bivariate finite mixture of regression models. These models offer some advantages since they have all the benefits of a finite mixture, allowing for unobserved heterogeneity and clustering effects, while the copula-based derivation can produce more flexible structures, including negative correlations and regressors. In this paper, the new approach is defined, estimation through an EM algorithm is presented, and then different models are applied to a Spanish insurance claim count database.

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**Inference for dependent error functional data with application to event-related**

P. 1100-1120

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**potentials**

Kun Huang, Sijie Zheng, Lijian Yang

**Abstract**

Estimation and testing is studied for functional data with temporally dependent errors, an interesting example of which is the event-related potential (ERP). B-spline estimators are formulated for individual smooth trajectories and their population mean as well. The mean estimator is shown to be oracally efficient in the sense that it is as efficient as the infeasible mean estimator if all trajectories had been fully observed without contamination of errors. The oracle efficiency entails asymptotically correct simultaneous confidence band (SCB) for the mean function, which is useful for making inference on the global shape of the mean. Extensive simulation experiments with various time series errors and functional principal components confirm the theoretical conclusions. For a moderate-sized ERP data set, multiple comparison is made by constructing paired SCBs among four different stimuli, over three components N450, N1, and N2 separately or simultaneously, leading to interesting findings.

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**Bayes factors for peri-null hypotheses**

P. 1121-1142

Alexander Ly, Eric-Jan Wagenmakers

**Abstract**

A perennial objection against Bayes factor point-null hypothesis tests is that the point-null hypothesis is known to be false from the outset. We examine the consequences of approximating the sharp point-null hypothesis by a hazy 'peri-null' hypothesis instantiated as a narrow prior distribution centered on the point of interest. The peri-null Bayes factor then equals the point-null Bayes factor multiplied by a correction term which is itself a Bayes factor. For moderate sample sizes, the correction term is relatively inconsequential; however, for large sample sizes, the correction term becomes influential and causes the peri-null Bayes factor to be inconsistent and approach a limit that depends on the ratio of prior ordinates evaluated at the maximum likelihood estimate. We characterize the asymptotic behavior of the peri-null Bayes factor and briefly discuss suggestions on how to construct peri-null Bayes factor hypothesis tests that are also consistent.

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**Estimation of poverty and inequality in small areas: review and discussion**

P. 1143-1166

Isabel Molina, Paul Corral, Minh Nguyen

**Abstract**

Never better said, a correct diagnosis is crucial for patient recovery. In the eradication of poverty, which is the first of the sustainable development goals (SDGs) established by the United Nations, efforts in the form of social aid and programs will be useless if they are not directed where they are most needed. Nowadays, monitoring the progress on the SDGs is even more urgent after the sanitary crisis, which is reversing the global poverty reduction observed since 1990 and, given that social development funds are always limited, managing them correctly requires disaggregated statistical information on poverty of acceptable quality. But reliable estimates on living conditions are scarce due to sample size limitations of most official surveys. Common small area estimation procedures supplement the survey data with auxiliary data sources to produce more reliable disaggregated estimates than those based solely on the survey data. We describe the traditional as well as recent model-based procedures for obtaining reliable disaggregated estimates of poverty and inequality indicators, discussing their properties from a practical point of view, placing emphasis on real applications and describing software implementations. We discuss results from recent simulation experiments that compare some of the unit-level methods in terms of bias and efficiency, under model- and design-based setups. Finally, we provide some concluding remarks.

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